REPORT

OPTICS

Breaking Lorentz reciprocity to overcome the time-bandwidth limit in physics and engineering

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A century-old tenet in physics and engineering asserts that any type of system, having bandwidth $\Delta\omega$, can interact with a wave over only a constrained time period Δt inversely proportional to the bandwidth ($\Delta t{\cdot}\Delta\omega \sim 2\pi$). This law severely limits the generic capabilities of all types of resonant and wave-guiding systems in photonics, cavity quantum electrodynamics and optomechanics, acoustics, continuum mechanics, and atomic and optical physics but is thought to be completely fundamental, arising from basic Fourier reciprocity. We propose that this "fundamental" limit can be overcome in systems where Lorentz reciprocity is broken. As a system becomes more asymmetric in its transport properties, the degree to which the limit can be surpassed becomes greater. By way of example, we theoretically demonstrate how, in an astutely designed magnetized semiconductor heterostructure, the above limit can be exceeded by orders of magnitude by using realistic material parameters. Our findings revise prevailing paradigms for linear, time-invariant resonant systems, challenging the doctrine that high-quality resonances must invariably be narrowband and providing the possibility of developing devices with unprecedentedly high time-bandwidth performance.

More than 100 years ago, K. S. Johnson introduced the concept of the now-ubiquitous quality factor (Q factor) to characterize the sharpness of a resonance (*1*, *2*). In that work, a practical way to characterize the quality of a resonant system was introduced by defining a unitless number $Q = \omega_0/\Gamma$, where ω_0 is the system's resonance frequency and Γ the decay rate of the wave energy (*1*, *2*). Since then, it has been understood that the higher the Q factor of a resonant system, the narrower its bandwidth—higher Qs lead to sharper resonances (*1*).

This notion, that high-quality (high-Q) resonances must invariably be narrowband, has not been challenged since Johnson's original work and pervades an extremely broad range of resonant and wave-guiding systems in physics and engineering (Fig. 1). Its justification arises from basic Fourier-reciprocity considerations (*3*–*5*): Inside any linear, passive (lossy and time-invariant) resonant system, e.g., in a cavity micro- or nanoresonator, the excited-wave amplitude $\alpha(t)$ will decay as $\alpha(t) \propto \cos(\omega_0 t) \times e^{-(1/2)\Gamma t}$, where Γ can be due to nonradiative (inelastic or dephasing) and/or radiative processes (coupling to the continuum of the surrounding medium). Hence, in the resonance approximation and in the usual underdamped regime ($\Gamma/2 << \omega_0$) (*3*, *4*), the intensity I in the frequency domain will be given by

$$I(\omega) \propto |\alpha(\omega)|^2 \propto \frac{\left(\frac{\Gamma}{2}\right)^2}{(\omega-\omega_0)^2 + \left(\frac{\Gamma}{2}\right)^2} \qquad (1)$$

from which it is immediately seen that $\Delta\omega$ around ω_0 is $\Delta\omega = (\omega_0 + \Gamma/2) - (\omega_0 - \Gamma/2) = \Gamma$. Thus, by definition, the bandwidth of the resonant system is the loss rate Γ. Any attempt to reduce the overall losses and, hence, store the wave for an increased time $\Delta\tau$ will automatically decrease $\Delta\omega$—a limitation that arises from simple time-harmonic considerations.

We show that this "fundamental" time-bandwidth limit characterizing resonant devices can be overcome by breaking Lorentz reciprocity, i.e., by conceiving (asymmetric) systems whose responses change when the source and the receiver are interchanged

$$\iiint_V \mathbf{J}_1\mathbf{E}_2 \mathrm{d}V \neq \iiint_V \mathbf{J}_2\mathbf{E}_1 \mathrm{d}V \qquad (2)$$

where $\mathbf{J}_1$ and $\mathbf{J}_2$ are two sources within a volume V generating the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$, respectively. Specifically, we introduce and analyze a realistic system that exceeds the time-bandwidth limit anticipated by the system's Q factor by orders of magnitude.

The time-bandwidth limitation is a completely general phenomenon, characterizing the storage capacity of all linear, time-invariant resonant and wave-guiding devices, from photonics to acoustics, cavity quantum electrodynamics and optomechanics, and atomic and molecular physics, as well as mechanical and structural systems (Fig. 1). It should not be confused with the mathematical time-bandwidth limit $\sigma_t^2\sigma_\Omega^2 \geq 1/4$, where σ_t^2 is the time variance of a signal $x(t) \in L_2(\Re)$ and σ_Ω^2 its frequency variance, i.e., with the uncertainty principle characterizing Fourier-integral pairs in signal analysis and communications (*6*) and which, among others, only has a lower bound. Although both limits often bear the same name, the time-bandwidth limit in physics and engineering characterizes the storage capacity of the devices themselves, not the mathematical Fourier properties of the respective signals. In addition to resonant physical devices outlined above, the physical time-bandwidth limit described in this study also arises in guiding structures, such as slow-light wave guides or bulk media (e.g., electromagnetically induced transparency in ultracold atomic gases) (*7*–*12*). Here, a number of works have shown that any such passive structure can support slow waves over a finite bandwidth $\Delta\omega$ inversely proportional to the group index n_g. Hence, a structure of fixed length L cannot delay a wave packet of bandwidth larger than $\Delta\omega$ by more than a time $\Delta\tau \sim n_g L/c$, where c is the speed of light in vacuum. In other words, the "delay-bandwidth product" $\Delta\tau{\cdot}\Delta\omega$, characterizing a linear, time-invariant slow-wave structure, has an upper limit C (*13*–*15*). This threshold is quite stringent: Depending on the specifics of the particular slow-wave structure, it can vary between $C \sim 10$ and 100, to within an order of magnitude (*7*–*10*, *13*–*15*), and cannot be broken by means of a nonlinear or gain mechanism, such as stimulated Raman or stimulated Brillouin scattering, because such fundamental effects as gain saturation, group-velocity, and attenuation dispersions make $\Delta\tau$ inversely proportional to a power of $\Delta\omega$—e.g., $\Delta\tau \sim \Delta\omega^{-\alpha}$, α = 2 or 3 (*7*, *9*, *14*)—an even stricter limitation. A further adverse consequence of the time-bandwidth limit in physics and engineering is that it constrains the response time of the above devices because the higher the Q factor of a system (i.e., the narrower the bandwidth), the longer it takes to respond to an external signal. However, a high Q factor is a prerequisite for high sensitivity (*16*). Thus, short response times and high sensitivity tend to counteract each other, and a compromise has to be found between the two. A well-known manifestation of this limitation concerns microfabricated quartz tuning forks, currently the most successful and widespread method for shear-force detection. With a Q factor at ambient conditions of the order of 10^3 to 10^4—necessary for probing interaction forces less than ~200 pN—the response time of a tuning fork, $\tau = 2\sqrt{3}Q/\omega_0$, is limited to being greater than ~300 ms (*16*), i.e., the scanning speeds are slow.

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To overcome the time-bandwidth limit by breaking Lorentz reciprocity, consider a wave s_+ impinging (either from a surrounding uniform medium or from a guiding structure) on a reciprocal system and exciting a mode of amplitude α inside it (Fig. 2A). The key idea is that although the basic Fourier-transform reciprocal relations do, in general, remain valid, they can be applied separately at the input and output ports of a system if it is asymmetric (nonreciprocal) in its transport properties, i.e., if Lorentz reciprocity is broken. The equation describing the time evolution of $\alpha(t)$ is (*17*)

$$\frac{d\alpha}{dt} = i\omega_0\alpha - \left(\frac{1}{\tau_0} + \frac{1}{\tau_{out}}\right)\alpha + \rho_{in}s_+ \quad (3)$$

where $1/\tau_0$ and $1/\tau_{out}$ are, respectively, the internal (owing, e.g., to dissipative losses) and out-coupling (owing, e.g., to radiative loss to the surrounding medium) decay rates, and ρ_{in} is the rate of in-coupling of energy from the s_+ wave to the resonant system. Here, following the standard convention of temporal coupled-mode theory, we assume that $|s_+|^2$ is normalized to the incident power, whereas $|\alpha|^2$ is normalized to the incident energy (i.e., the units of $|s_+|$ are $\sqrt{2W/s}$) (*17*). The rate of in-coupling of energy into the resonant system ρ_{in} is proportional to the Δω of the system ($\pi\rho_{in} \leftrightarrow \Delta\omega$) (*17*, *18*) [see also (*19*)], whereas the lifetime Δτ of the excited mode is, as shown from Eq. 3, $\Delta\tau = 1/(1/\tau_0 + 1/\tau_{out}) \approx \tau_{out}$ because we normally operate in the overcoupled (underdamped) regime where the rate of energy escape from (and energy coupled into) the excited resonant system is greater than the rate of internal dissipation, $2/\tau_{out} >> 2/\tau_0$. The key point is that, because of time-reversal symmetry, it can be shown rigorously that the in-coupling rate ρ_{in} is always tied to the out-coupling rate ρ_{out} through the exact relation (in units of $|\alpha|$) (*17*):

$$|\rho_{in}| = \frac{2}{\tau_{out}} \quad (4)$$

Thus, the product between the system's bandwidth and the wave-system interaction time (lifetime) Δτ is always, for reciprocal systems, of the order of $\Delta\omega\Delta\tau \leftrightarrow \pi|\rho_{in}|\tau_{out} \sim 2\pi$, i.e., we recover the aforesaid physical time-bandwidth limitation, in which Δω and Δτ characterizing a resonant or guiding device are reciprocally related. However, if Lorentz reciprocity is by some means broken in this passive, linear, and time-invariant resonant system, $|\rho_{in}|$ and τ_{out} can become completely decoupled, in which case the product ΔωΔτ (or, equivalently, $|\rho_{in}|\tau_{out}$) can be engineered at will and take on arbitrarily large values—i.e., in such a case we can exceed the conventional time-bandwidth limit by an arbitrarily large factor.

Consider a heterostructure made of a dielectric layer (silicon, Si) bounded asymmetrically by a gyroelectric semiconductor (indium antimonide, InSb) on the bottom and a metal layer (silver, Ag) on the top (Fig. 2B). Lorentz reciprocity in this linear, passive, and time-invariant system can be broken by applying a static magnetic field B_0 in the $-y$ direction (*20*–*22*), causing a precession of the electron magnetic dipole moments in the semiconductor with a frequency $\omega_c = eB_0/m*$ (where e and $m*$ are the charge and effective mass of the electrons, respectively). A small ac magnetic field propagating along the heterostructure also causes a precession of the semiconductor electrons' dipole moments around the B_0 ($-y$) axis at the frequency of the ac field. The interaction of the ac field with the semiconductor is thus, overall, captured by the following asymmetric permittivity tensor (*23*)

$$\varepsilon = \varepsilon_0\varepsilon_\infty \begin{bmatrix} \varepsilon_1(B_0) & 0 & i\varepsilon_2(B_0) \\ 0 & \varepsilon_3 & 0 \\ -i\varepsilon_2(B_0) & 0 & \varepsilon_1(B_0) \end{bmatrix} \quad (5)$$

where $\varepsilon_1 = 1 - (\omega + i\nu)\omega_p^2/\{\omega[(\omega + i\nu)^2 - \omega_c^2]\}$; $\varepsilon_2 = \omega_c\omega_p^2/\{\omega[(\omega + i\nu)^2 - \omega_c^2]\}$; $\varepsilon_3 = 1 - \omega_p^2/[\omega(\omega + i\nu)]$, with the plasma frequency of InSb taken to be $\omega_p = 4\pi \times 10^{12}$ rad/s ($f_p = 1/T_p = 2$ THz); $\varepsilon_\infty = 15.6$; $\omega_c = 0.2\omega_p$; and $B_0 = 0.2$ T [where ε, permittivity; v, the (permittivity) loss factor; p, plasma; T, (time) period]. For the other two layers, we take $\varepsilon_{Si} = 11.68$ and $\varepsilon_{Ag} = 1 - \omega_{pe}^2/[\omega(\omega + i\omega_\tau)]$, with $\omega_{pe} = 1.367 \times 10^{16}$ rad/s and $\omega_\tau = 2.733 \times 10^{13}$ rad/s (*20*, *21*, *23*). Because of the application of the external magnetic bias, the heterostructure supports one-way edge (magnetoplasmon) modes, robust against surface imperfections and roughness, whose dispersion relation is governed by (*20*, *21*, *23*)

$$\tanh(\alpha_d d) = -\frac{\alpha_s + \frac{\varepsilon_2}{\varepsilon_1}k + \frac{\varepsilon_V\alpha_m}{\varepsilon_m}}{\left(\alpha_s + \frac{\varepsilon_2}{\varepsilon_1}k\right)\frac{\varepsilon_r\alpha_m}{\varepsilon_m\alpha_r} + \frac{\varepsilon_V\alpha_d}{\varepsilon_r}} \quad (6)$$

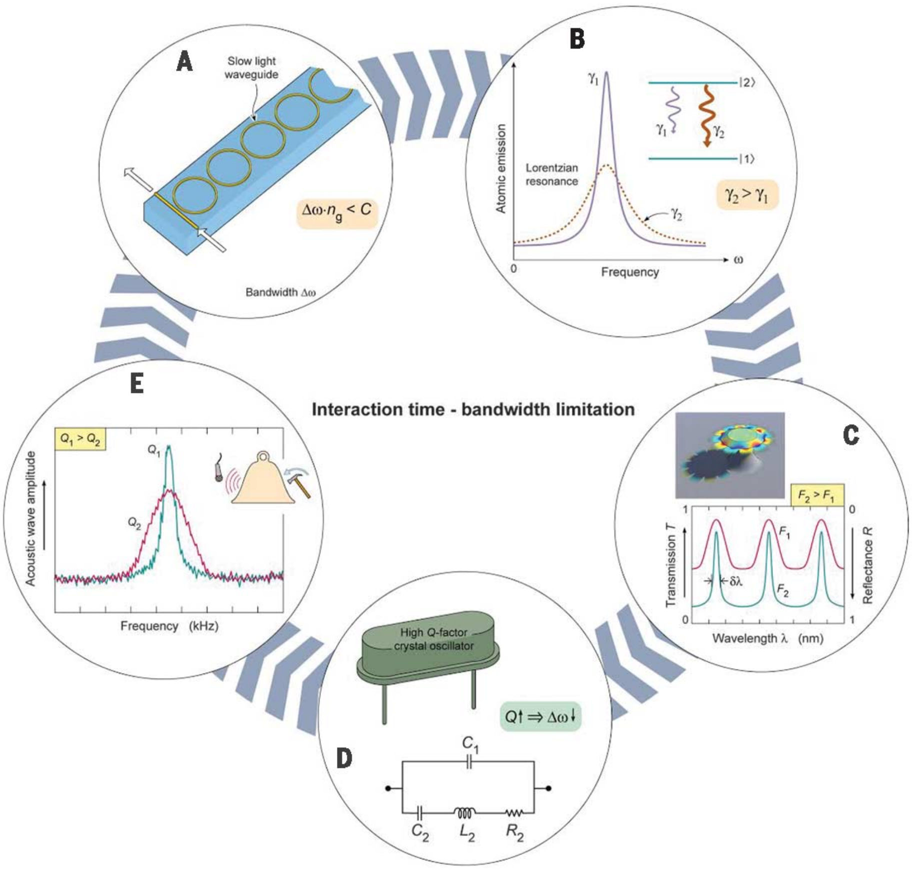


Fig. 1. The "fundamental" time-bandwidth limit, in various forms, in reciprocal systems in physics and engineering. (**A**) In all types of slow-light wave guides, the attained delays Δt are inversely proportional to the guide's bandwidth, Δω ($\Delta t \propto \Delta\omega^{-1}$) or, even more severely, to a power of it (e.g., $\Delta t \propto \Delta\omega^{-\alpha}$, α = 2 or 3) (*7*–*15*). (**B**) In atomic and molecular physics, the linewidth γ of an atomic transition is inversely proportional to the decay rate arising from dephasing and inelastic or spontaneous-emission processes (*3*). Likewise, the time required to perform the transformation $|\Psi_i\rangle \rightarrow |\Psi_f\rangle = e^{-iHt}|\Psi_i\rangle$, where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are two orthogonal states and H is the (time-independent) Hamiltonian, is $\tau_{tr} \sim |(E_f - E_i)/\hbar|^{-1}$, where E_f and E_i are the corresponding eigenvalues of H (*24*). (**C**) In all types of (dielectric or plasmonic) cavity resonators, higher finesses F result in narrower resonance bandwidths (*3*–*5*, *16*–*18*). (**D**) In crystal (quartz) oscillators; piezoelectic, micro-/nanomechanical, or elastic systems; and energy-harvesting devices, the response times are directly proportional to the system's Q factor, $\tau_{rsp} \propto Q$. Higher Q factors lead to enhanced sensitivities but also to larger response times (*16*). (**E**) In acoustic devices and systems, such as in ultrasound, elastic-wave, or wave-modulation spectroscopies, increased quality factors give rise to narrower spectral responses.

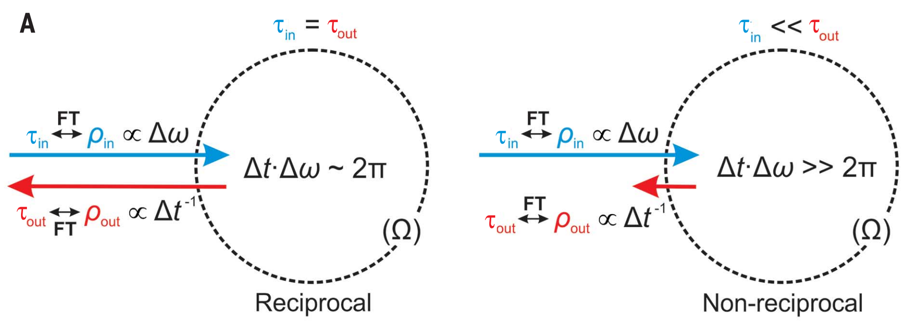
where $\alpha_d = \sqrt{k^2 - \varepsilon_r k_0^2}$; $k_0 = \omega/c$ is the vacuum wave number; $\varepsilon_r = \varepsilon_{Si}$ and $d = 0.08\lambda_p(\lambda_p = 2\pi c/\omega_p)$ are the relative permittivity and thickness of the Si layer, respectively; $\alpha_s = \sqrt{k^2 - \varepsilon_V k_0^2}$; $\varepsilon_V = \varepsilon_\infty(\varepsilon_1 - \varepsilon_2{}^2/\varepsilon_1)$ is the Voigt permittivity; and $\alpha_m = \sqrt{k^2 t - \varepsilon_m k_0^2}$, where ε_m is the relative permittivity of Ag.

Upon solving Eq. 6, we plot in Fig. 2C the band structure of this type of surface state, showing clearly that the band diagram is asymmetric with respect to the wave vector $-k$ axis, giving rise to a frequency region where no backward-propagating ($k < 0$) states exist (breaking of Lorentz reciprocity). For a carefully designed structure, that region can be made to be below the continuous band(s) of the bulk modes in the semiconductor and above the band associated with surface states at the semiconductor-metal interface. Thus, in that frequency region, complete unidirectional propagation (CUP) is rigorously attained: An excited edge state can propagate strictly only in the forward (positive z, k) direction and cannot be back-reflected or couple either to bulk modes in the semiconductor or to semiconductor-metal surface states. The two frequencies, ω_{CUP}^- and ω_{CUP}^+, bounding the CUP region (see Fig. 2C) can be identified analytically from Eq. 6 by letting $|k| \to \infty$ [see (*19*)]

$$\omega_{CUP}^{\pm} = \frac{1}{2}\left(\sqrt{\omega_c{}^2 + 4\omega_p{}^2 \frac{\varepsilon_\infty}{\varepsilon_\infty + \varepsilon_r}} \pm \omega_c\right) \qquad (7)$$

from which we see that the bandwidth of the CUP region is simply $BW_{CUP} = \omega_c(= eB_0/m*)$.

Figure 3A illustrates successive snapshots from full-wave simulations of the propagation of a pulse, whose bandwidth is within the CUP region, along the heterostructure shown in Fig. 2B. The structure is terminated in the z direction by the Ag cladding (which also covers the end of the heterostructure), creating an impenetrable barrier for the pulse along z. Because in that frequency region there are no surface modes allowed at the Ag-InSb interface and the pulse cannot scatter to bulk modes inside InSb or to backward modes in the $-z$ direction (see Fig. 2C), the pulse eventually localizes near the Si-Ag interface, where it decays with time until it is completely absorbed (right-hand graph of Fig. 3A). As seen in Fig. 3A, initially, at t = 15 ps (= $30T_p$), the pulse broadens (because of dispersion) to a longitudinal length of $d_i^l \cong$ 215.5 μm [and a transverse size of $d_i^T \cong$ 12.1 μm; see also fig. S1, A and B, (*19*)], but when it reaches the rightmost end, it gives rise to a strongly localized plasmonic resonance. At t = 50 ps (= $100T_p$), the pulse is spatially compressed to a deep-subwavelength spot of $d_f^l \cong$ 0.165 μm (and $d_f^T \cong$ 0.02 μm), i.e., it is spatially squeezed by a factor of ~0.79×10^6 in two dimensions, while its peak intensity is enhanced by a factor of ~10^3 [see also fig. S1, D and E, (*19*)] (where T, transverse; p, period; l, length in the longitudinal direction; i, initial; f, final). The localized field thus behaves exactly as if it were confined inside a subwavelength, "zero-dimensional" (*24*) cavity resonator perfectly matched to the incident-wave medium: It is confined in a specified region of space—where it was in-coupled without reflections, decaying with time inside (but not propagating within or escaping from) this region—and with the field amplitude being dramatically enhanced inside this zero-dimensional cavity. Figure 3B further shows that the so-trapped field can be released on demand by reversing the direction of the external magnetic bias B_0 (B_0 = 0.2 T → B_0 = −0.2 T) at any point while the field is localized.



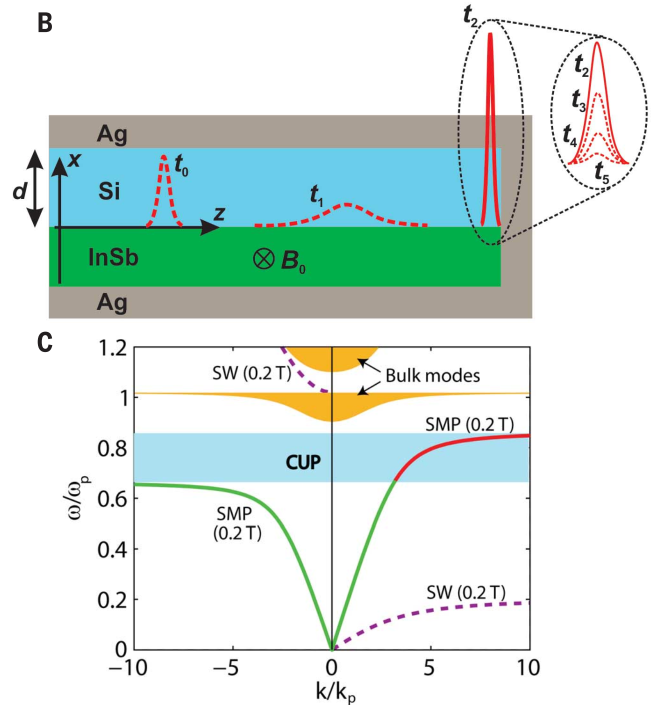


Fig. 2. Concept and structure for overcoming the time-bandwidth limit. (**A**) In a reciprocal system (left), the rate ρ_{in} with which energy enters the system is equal to the rate ρ_{out} with which energy exits the system. This leads to the interaction time being inversely proportional to the system's bandwidth, $\Delta t \propto \Delta\omega^{-1}$. By contrast, in a nonreciprocal system (right) the in-coupling time, τ_{in}, can be much shorter than the out-coupling time, $\tau_{in} << \tau_{out}$ (or, equivalently, $\rho_{in} >> \rho_{out}$), leading to an interaction time Δt that can be completely decoupled from the bandwidth $\Delta\omega$. Consequently, the product $\Delta t \cdot \Delta\omega$ can now take on arbitrarily large values—much larger than the standard 2π limit. In both cases, the Fourier-transform reciprocity, relating τ_{in} (τ_{out}) to ρ_{in} (ρ_{out}) and to $\Delta\omega$ (Δt^{-1}), is always valid, but is applied separately to the input and the output of the system. (**B**) Schematic illustration of the nonreciprocal configuration used for breaking the time-bandwidth limit. It consists of a Si layer sitting on top of a gyroelectric semiconductor, InSb. The two layers are bounded by Ag on three sides. The external, static magnetic field B_0 is applied in the $-y$ direction. A pulsed magnetic-current source excites a surface magnetoplasmon, which propagates unidirectionally, without back-scattering or back-reflections, from left to right, all the way until the rightmost Si-Ag interface where it is spatially compressed, greatly enhanced in amplitude, and robustly localized (compare to Fig. 3A). There are no propagating states allowed in the $-z$ direction, inside InSb, or along the InSb-Ag interface [see also (C)]. Once localized at the rightmost end, a pulse can only decay with time owing to dissipative losses, exactly as if it were confined inside a passive, zero-dimensional (*24*) cavity resonator (right inset). (**C**) Band diagram of the structure of (B) for the case where the external magnetic field is applied in the $-y$ direction (B_0 = 0.2 T). Shown are, for both positive and negative longitudinal wave vectors k, the dispersion curve of the herein studied surface magnetoplasmons (SMPs), the surface wave at the InSb-Ag interface (SW), and the region of the bulk modes in InSb. The area shaded in blue indicates the band region where CUP of the considered surface magnetoplasmon is attained. The part of the SMP dispersion curve inside that region is indicated in red.

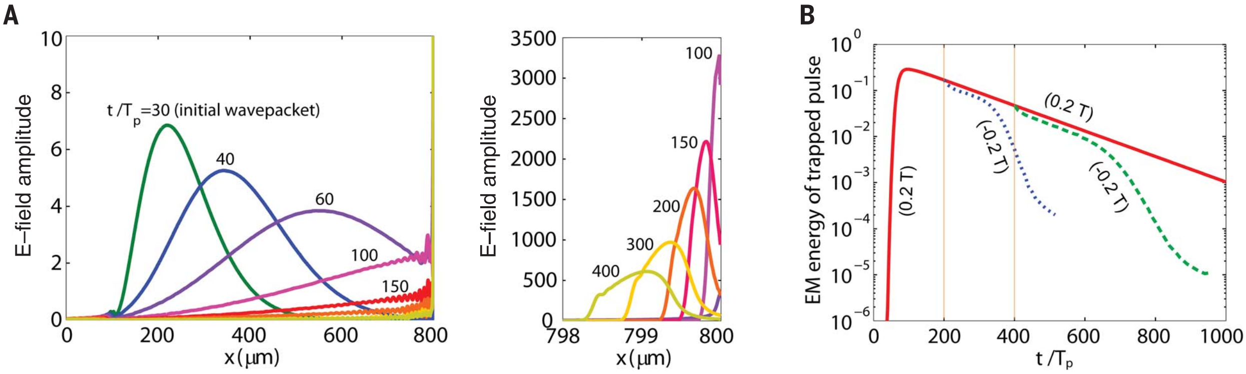


Fig. 3. Open-cavity localization and overcoming the "fundamental" delay-time–bandwidth limit. (**A**) One-dimensional snapshots of the propagation of a pulse of central frequency 1.5 THz and bandwidth 0.2 THz, exciting a one-way surface magnetoplasmon in the heterostructure of Fig. 2B, at successive time instants. Shown on the right are close-up views around the localization region at the rightmost end of the heterostructure of Fig. 2B. Note the change in the scale of the vertical axis and the dramatic ensuing field enhancement upon entering that zero-dimensional open-cavity region. E-field, electric field. (**B**) Electromagnetic energy as a function of time in an imaginary box surrounding the pulse's localization region. When the external static magnetic field B_0 = 0.2 T is constantly in the −y direction, the energy of the localized pulse inside the box simply decays with time (red curve) owing to dissipative losses, as expected [and as shown in (A)]. By contrast, when the direction of the B_0 field is suddenly reversed (B_0 = −0.2 T), the pulse may now escape in the backward (−z) direction (compare to Fig. 2C), as a result of which the wave energy inside the box rapidly diminishes. Two examples of this are shown here, one at t = 200T$_p$ (blue dotted line) and another at t = 400T$_p$ (green dashed line), demonstrating storage times of up to ~400T$_p$.

A somewhat reminiscent light-trapping, storage, and releasing scheme also exists, e.g., for ultraslow and stored light in atomic electromagnetically induced transparency (EIT) (*8*, *11*, *12*) but with the fundamental difference being that therein the bandwidth is narrow (*25*) and/or the attained storage times are inversely proportional to the bandwidth (or to a power of it) (*7*, *9*, *13*–*15*). Figure S2 (*19*) shows how in this linear, time-invariant system the whole broad spectrum of the pulse is progressively stored in its trapping region.

Because of the above Lorentz reciprocity–breaking characteristics, the rates of in-coupling (ρ_{in}) and out-coupling (ρ_{out}) of energy in this open cavity are not equal: Whereas ρ_{in} is proportional to the system's in-coupling bandwidth ($\rho_{in} \propto \Delta\omega_{in}$), the out-coupling rate tends to zero ($\rho_{out} = 1/\tau_{out} \rightarrow 0$) because the light wave cannot radiatively escape from the region it is confined in. Thus, on the basis of our previous analysis, we expect that in this system the interaction time $\Delta\tau = 1/(1/\tau_0 + 1/\tau_{out}) \approx \tau_0$ and the resonant bandwidth $\Delta\omega$ should be completely decoupled, not inversely proportional as in all conventional (reciprocal) resonant and wave-guiding systems. In other words, we expect that our system can be extremely broadband even in the limit of ultrahigh Q factors where the total losses may tend to zero and the storage times to infinity ($\Delta\tau \rightarrow \infty$).

To demonstrate that $\Delta\tau$ and $\Delta\omega$ are independent of one another, Fig. 4 summarizes the results of successive full-wave simulations for the cases where (i) the loss rate ν is progressively increased, but B_0 remains constant and (ii) B_0 progressively increases, but ν remains constant. We see from Fig. 4, A and B, that while v is increased, the total optical losses of the system also progressively increase as expected, but the bandwidth of the effective cavity remains constant in all cases, $\Delta\omega \approx 2.5$ THz, unaffected by the gradually increased loss rate. Even in the case where an extremely low-loss InSb film ($\nu/\omega_p = 10^{-4}$) with realistic material parameters is used [e.g., electron density $N_e = 1.1 \times 10^{16}$ cm^{-3}; see (*26*) and (*19*)], we find that whereas the energy decay rate Γ is ~10^9 s^{-1} and therefore the bandwidth $\Delta\omega$ should conventionally be anticipated to be small, ~10^{-3} THz ($\Delta\omega = \Gamma$, see discussion immediately after Eq. 1), the actual bandwidth of the nonreciprocal zero-dimensional cavity at the rightmost end of our structure is still large and ~2.5 THz—more than three orders of magnitude above the fundamental time-bandwidth limit of reciprocal (linear and passive) systems. Furthermore, we find that the pulse is seamlessly in-coupled to the localization point (whereas for any reciprocal, lossless resonant system, the in-coupling time would have tended to infinity for progressively smaller losses), where it is rigorously confined (Fig. 3A and leftmost parts of Fig. 4, A and B). Thus, the performance of this system, both in terms of bandwidth and response time, exceeds that of any standard reciprocal system (*4*, *5*, *16*–*18*, *23*, *24*) by orders of magnitude.

For case (ii), where the external static magnetic field B_0 progressively increases, Fig. 4, C and D, shows that the bandwidth of the zero-dimensional cavity increases accordingly (by 100%), as expected from Eq. 7, but the optical losses (and hence the storage times) remain approximately constant, increased only by ~5%—unaffected by the bandwidth increase. The small increase in the total optical losses that we observed in our simulations for this latter case is because the slope of the band (i.e., the pulse's group velocity) reduces with increasing B_0, leading to higher overall optical losses (*27*). We see from Fig. 4, C and D, that, in this case too, the nonreciprocal cavity is above the fundamental time-bandwidth limit of conventional (reciprocal) resonant systems by more than two orders of magnitude. The results presented in Fig. 4, therefore, convincingly show that in this system, the interaction time (lifetime) $\Delta\tau$ and the bandwidth $\Delta\omega$ are independent and decoupled of one another, owing to the breaking of Lorentz reciprocity ($\rho_{in} >> \rho_{out}$), giving rise to an, in principle, unlimited time-bandwidth performance—i.e., to breaking of the Q-factor limit in the sense that $\Delta\tau$ and $\Delta\omega$ are not inversely proportional to one another anymore [although Fourier uncertainty (*6*) is still obeyed when considered separately at the input and output ports, as shown in Fig. 2A and discussed above].

Finally, in existing, reciprocal ultraslow- and stored-light configurations (e.g., those exploiting dark states in EIT or in coherent population trapping), the storage time is fundamentally inversely proportional to the system's bandwidth or to a power of it (*7*–*15*). In contrast, in the present nonreciprocal scheme, the storage time is solely determined by the loss rate (which, as shown in Fig. 4, is decoupled from the bandwidth) and/or the time until which we switch off the external magnetic field, releasing the localized pulse (Fig. 3B). Because both of these parameters (loss rate and duration of B_0 being "on") are, here, completely independent of the system's bandwidth, the attained delay-bandwidth products can now, in principle, become arbitrarily large. For instance, Fig. 3B demonstrates storage times of up to ~400T$_p$ for a pulse of bandwidth 0.2 THz, whereas conventionally, for reciprocal-guiding structures, the anticipated maximum

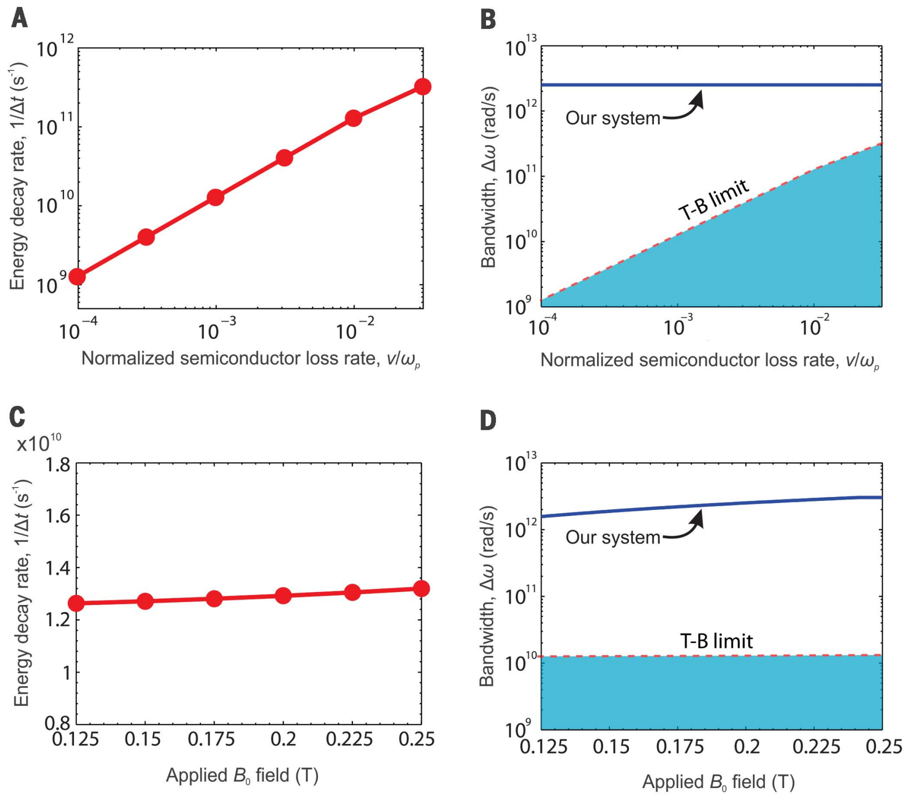


Fig. 4. Decoupling of interaction time and bandwidth and overcoming the Fourier-reciprocity ($\Delta\omega \sim \Delta t^{-1}$) limit. (**A**) and (**B**) For increasing scattering losses v in the semiconductor, the decay rate of the localized pulse's energy progressively increases (A), signifying reduced interaction (storage) times, as expected, whereas in all cases the bandwidth $\Delta\omega$ of the one-way effective cavity remains constant (B) (blue line). The energy decay rate sets the fundamental bandwidth limit [$\Delta\omega = \Gamma \sim \Delta t^{-1}$, time-bandwidth (T-B) limit] characterizing conventional, reciprocal systems and which is here broken by more than three orders of magnitude, as shown in (B). By contrast, for increasing values of the static magnetic bias B_0, the energy decay rate remains approximately constant (**C**), whereas the bandwidth of the zero-dimensional open cavity progressively increases (**D**) (blue line) [see also fig. S3B for further clarity (*19*)]. Note from (D) that in this case, the bandwidth of the zero-dimensional cavity is more than two orders of magnitude above the fundamental time-bandwidth limit of reciprocal resonant devices—i.e., larger than the energy decay rate by more than two orders of magnitude. All the results shown here have been obtained from full-wave and analytic calculations, as detailed in (*19*). In both (B) and (D), the dashed red lines are the solid red lines shown in (A) and (C), respectively.

delay and storage times would be (*7–15*) $\Delta t_{max} \sim (\Delta\omega)^{-1}$ or less, i.e., $\Delta t_{max} \sim 5$ ps = $10T_p$. Thus, our nonreciprocal device is above the conventional delay time–bandwidth limit of state-of-the-art slow-light systems by more than a factor of 40.

The consequences of our findings carry over to all resonant and wave-guiding systems in physics and engineering where the above time-bandwidth limit appears in disguise, including subdiffraction imaging systems (where there is always a trade-off between spatial and temporal resolution) (*28*) and broadband invisibility cloak devices (where there is a trade-off between scattering reduction and broadband operation) (*29*, *30*). On a more basic level, our results reveal that the time-bandwidth and *Q*-factor limits characterizing the storage capacity of (passive, linear) guiding and resonant systems in physics and engineering are not as "fundamental" as has conventionally been thought and can be broken to an arbitrarily large degree, so long as Lorentz reciprocity is broken in those systems. To this end, further means of breaking unidirectionality (*22*), such as parity-time–symmetry media (*31*) or topological insulators (*32–34*), might also be of interest. We believe that it is now possible to design ultrahigh-*Q* resonant systems in atomic, optical, and condensed matter physics, as well as in mechanical and electrical engineering, with unprecedentedly high bandwidths and ultrafast response times, in addition to ultraslow- and stopped-light systems with unusually high delay-bandwidth products, for a wide range of applications in those fields (*3*, *8–10*, *16–18*, *23*, *24*).

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SUPPLEMENTARY MATERIALS

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Figs. S1 to S3
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Breaking Lorentz reciprocity to overcome the time-bandwidth limit in physics and engineering

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Resonant systems with high bandwidth

The performance of an active system, whether it is optical, electrical, or mechanical, is often described by its quality (*Q*) factor. Typically, one learns the rule that the higher the *Q* factor, the sharper the resonance —that is, the bandwidth of the device is reduced. Tsakmakidis *et al.* show that this is indeed the case, but only for symmetric systems. However, for the case of asymmetric (or nonreciprocal) systems, the rule need not be obeyed. They show theoretically that the more asymmetric a system with high *Q* is, the wider the bandwidth can be. The effect raises the prospect of designing high- *Q* devices operating over large bandwidths.

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