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# Nonreciprocal cavities and the time-bandwidth limit: comment

KOSMAS L. TSAKMAKIDIS,[1,*,†] YUN YOU,[2] TOMASZ STEFAŃSKI,[3] AND LINFANG SHEN[2,†]

[1]Solid State Physics section, Department of Physics, National and Kapodistrian University of Athens, Panepistimioupolis, GR – 157 84, Athens, Greece
[2]Department of Applied Physics, Zhejiang University of Technology, Hangzhou 310023, China
[3]Gdańsk University of Technology, Faculty of Electronics, Telecommunications and Informatics, ul. G. Narutowicza 11/12, 80-233 Gdańsk, Poland
*Corresponding author: ktsakmakidis@phys.uoa.gr

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**In their paper in Optica 6, 104 (2019), Mann *et al*. claim that linear, time-invariant nonreciprocal structures cannot overcome the time-bandwidth limit and do not exhibit an advantage over their reciprocal counterparts, specifically with regard to their time-bandwidth performance. In this Comment, we argue that these conclusions are unfounded. On the basis of both rigorous full-wave simulations and insightful physical justifications, we explain that the temporal coupled-mode theory, on which Mann *et al*. base their main conclusions, is not suited for the study of nonreciprocal trapped states, and instead direct numerical solutions of Maxwell's equations are required. Based on such an analysis, we show that a nonreciprocal terminated waveguide, resulting in a trapped state, clearly outperforms its reciprocal counterpart; i.e., both the extraordinary time-bandwidth performance and the large field enhancements observed in such modes are a direct consequence of nonreciprocity.** 

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The paper by Mann *et al*. [1] investigates a time-invariant, unidirectional waveguide interacting with a cavity [Fig. 1(a)], concluding that the behavior of the cavity remains unchanged by the presence of the waveguide. This conclusion is then generalized to stating that time-invariant nonreciprocal systems cannot overcome the time-bandwidth (T-B) limit. These assertions appear to conflict with our previous work on a terminated unidirectional waveguide [Fig. 1(b)], which we deployed to report that large (by a factor of ∼1000) T-B violations in linear, time-invariant nonreciprocal systems could be achieved [2].

However, in this Comment we will show that the discrepancy in the conclusions of these two works stems entirely from the nature of the selected tool of analysis used in Ref. [1]: the main conclusions reached by Mann *et al*. concerning the T-B performance of linear time-invariant systems were on the basis of a temporal coupled-mode theory (TCMT), thereby relying on an analytic *TCMT approximation* ansatz [1,3], whereas all the numerical results of Ref. [2] reporting large T-B violations in the same systems were based on full-wave finite-difference time-domain (FDTD) simulations [2]. We will outline that the TCMT used in Ref. [1] is not suited for the study of the structure reported in Ref. [2], which includes a trapped state [blue shading in Fig. 1(b)], and that the extraordinary T-B performance observed in [2] is a direct consequence of the nonreciprocal nature of the device.

To begin, we recall that TCMT [3] describes the evolution of a field inside a cavity according to the following equation:

$$\frac{da}{dt} = i\omega_0 a - (\gamma_i + \gamma_r)a + \kappa_{\text{in}} s_+, \tag{1}$$

where $a$ is the field amplitude inside the cavity, $\omega_0$ is the resonance frequency dictated by the cavity (or cavity mode), $\gamma_i$ and $\gamma_r$ are the intrinsic and radiative loss rates, respectively, $|s_+|^2$ is the power incident onto the cavity from an external system, e.g., a waveguide, and $\kappa_{\text{in}}$ is the coupling coefficient between that external system and the cavity.

Equation (1) is satisfied when the field inside the cavity takes the form

$$a(t) = a_0 e^{i\omega_0 t - t(\gamma_i + \gamma_r)}, \tag{2}$$

where,

$$a_0 = \frac{\kappa_{\text{in}} s_+}{i(\omega_0 - \omega) - \gamma_i - \gamma_r}. \tag{3}$$

Note, here, that Eq. (3) has the exact same form as Eq. (4) of Ref. [1]. We see that the in-coupling coefficient, $\kappa_{in}$, determines the amplitude of the field inside the cavity; the frequency-dependent behavior is solely dependent on the denominator. Specifically, the cavity has a half maximum when $|\omega - \omega_0| = \gamma_i + \gamma_r$. Therefore, the bandwidth $\Delta\omega$ of an ordinary closed cavity is directly linked to the energy decay rate ($\Delta\omega = \gamma_{\text{tot}} = \gamma_i + \gamma_r$) [2], and it is thus inversely proportional to the decay (storage) time, resulting in the well-known time-bandwidth limit [2]. Note that this depends only on the cavity properties, independent of the nature of the feeding waveguide (reciprocal or nonreciprocal).

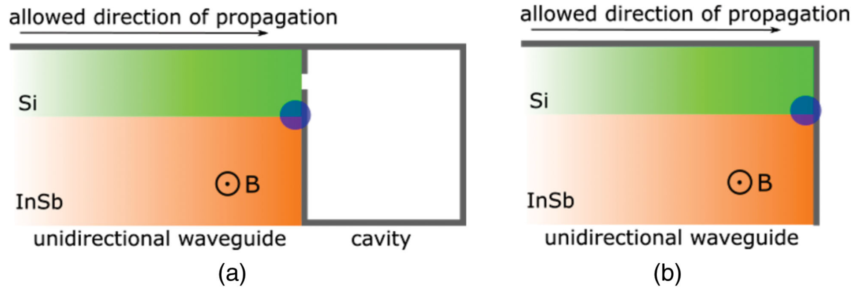


**Fig. 1.** Two systems considered in this Comment, both featuring a unidirectional magneto-optic waveguide (red/green region) bounded by metal layers (gray) and exhibiting a wedge mode (also called trapped state or open cavity) at the termination (blue shading). (a) Mann *et al.*'s case in which the waveguide couples to a lossless metallic cavity. The gap in the metal (slit) acts as both the input and output ports of the cavity, which is therefore not strictly unidirectional. (b) Same waveguide, but simply terminated by a plasmonic metal layer [2].

However, intrinsic to this TCMT description are several key assumptions and approximations, making this approach inapplicable to nonresonant trapped states (e.g., the blue shaded regions in Fig. 1). As shown below, the conclusions drawn by Mann *et al*. based on such a TCMT analysis cannot, therefore, be extended to the trapped state. We note that these states are refered to as "wedge mode," "open cavities," or "trapped states" in varying works, and for the remainder of this Comment we shall use the latter term.

The standard form of TCMT, analyzed in detail in [1] both for reciprocal and nonreciprocal feeding, assumes that a cavity mode must be a confined, oscillatory mode with a well-defined, single resonance frequency $\omega_0$; that is, it describes resonances peaked at a single frequency $\omega_0$ [owing to the '$i\omega_0 a$' term in Eq. (1)]. Such an assumption for a resonance, i.e., that it should have a well-defined single peak (at an '$\omega_0$'), though reasonable in ordinary cavities, does not describe key features of the trapped state of Fig. 1(b)—a point that is now clarified and proved below, with the aid of Fig. 2. Here (in Fig. 2), we apply TCMT to the system [1,3,4], with $\omega_0$ being the central frequency of the complete unidirectional propagation (CUP) region [2], and compare these calculations with direct numerical solutions of Maxwell's equations, obtained through FDTD simulations of exactly the same structure and conditions. Specifically, in both cases, we use the same lossy structure (with $\upsilon = 5 \times 10^{-4}\omega_p$ [2], characterizing the losses of InSb) and eliminate cavity back-reflection(s) [3,4] in both calculation approaches. We see from Fig. 2 that the FDTD calculation predicts a broad and *flat-top* (no single-peak/plateaued) response, while TCMT predicts a narrowband response peaked, as always in that theory, at a single frequency $\omega_0$.

Which one is the physically correct result? Clearly, the physically correct result is that of the FDTD method because within the CUP region the trapped state cannot radiatively escape its localization region, and therefore it can only 'escape' the system nonradiatively, by eventually being 100% absorbed *within the entire CUP region*—as shown by the red curve in Fig. 2. As such, TCMT is, evidently, not applicable to the trapped state, whose bandwidth is here, as calculated from Fig. 1, in fact $\sim$ 1000 broader than that predicted from TCMT. This factor ($\sim$1000) is in fact the degree to which the T-B limit is overcome in the structure of Fig. 1(b), as was reported in Fig. 4 of Ref. [2], precisely because, as was also outlined above, the TCMT always gives rise to T-B-limited resonances [2]. In other words, one cannot deploy (at least, the standard form of) a TCMT approximation, which inherently gives rise to T-B-limited resonaces, to investigate whether a structure might violate (or not) the T-B limit—the result will always be negative, owing to the inherent 'structure' (ansatz) of that theory. For such an analysis, *ab initio* full Maxwell solvers are required, as Fig. 2 above shows, and as Ref. [2] has reported. Note that, interestingly, here, TCMT fails even in the low-loss regime where it is usually successfully applied (e.g., in silicon photonics or in dielectric photonic crystals; cf. lossless structure studied in Ref. [1]); i.e., the failure arises not from the second term on the right-hand side of Eq. (1), but from the first term ('$i\omega_0 a$') on the same side of Eq. (1)—a feature that, to our knowledge, has not been identified in the past, since it does not normally arise in ordinary (non-topological) resonant structures.

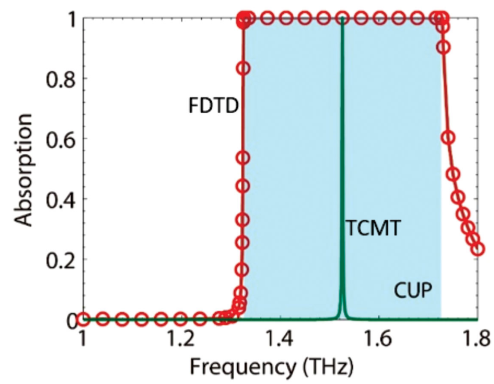


**Fig. 2.** Absorption resonance of the trapped state in the nonreciprocal structure of Fig. 1(b), as calculated through FDTD simulations (red) and a temporal coupled-mode theory analysis (green)—in both cases, for exactly the same lossy structure and reflectionless scenario. From the FDTD simulations it is found that at the center of the CUP region $\gamma_i \sim 3.0614 \times 10^9$ rad/s [cf. Eq. (1)]. In the temporal coupled-mode theory calculation, the $\gamma_r$ term [cf. Eq. (1)] is also set equal to $\sim 3.0614 \times 10^9$ rad/s, to allow for the absence of back-reflection(s) from the terminating cavity [3,4]. For the FDTD calculation, the shown absorption profile was calculated in the near field of the excited non-self-sustained [5] bulk plasmon [6] of the terminating (planar) Ag particle, which here itself acts as an open cavity (see also main text), similarly to standard calculations of absorption profiles of plasmonic particles in nanoplasmonics [6].

Mann *et al*. have also taken the in-/out-coupling rates in a nonreciprocal cavity shown in Fig. 2(a) of Ref. [2] (indicated therein, respectively, with cyan/red colors) to represent the *total* in-/out-coupled energy rate (power), whereas in fact those rates only refer to the *radiative* part of the power, as was explained in Ref. [2] (cf. '$\tau_{out}$' in Fig. 2(a) of Ref. [2] with '$\tau_{out}$' in Eq. (3) of Ref. [2]; i.e., the red arrow in both panels of Fig. 2(a) of Ref. [2] is associated with

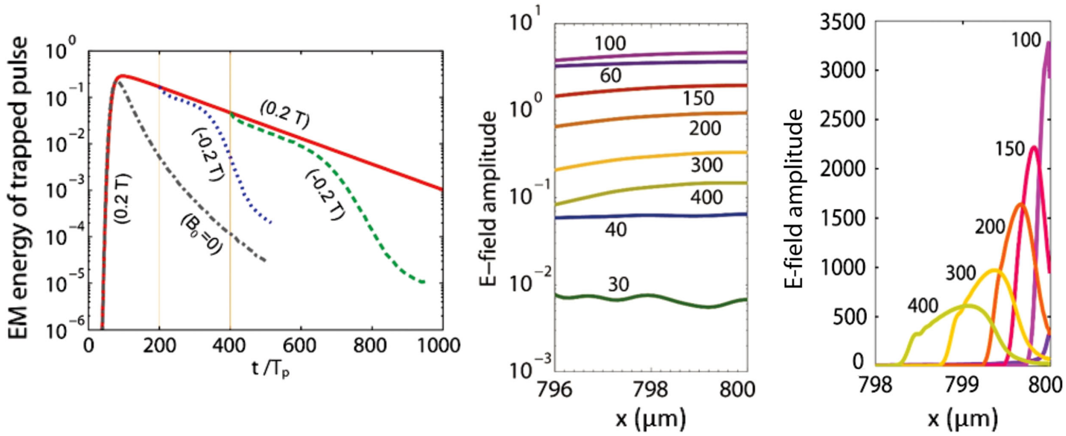


**Fig. 3.** Reciprocal versus nonreciprocal open cavities. (a) Shown is the time evolution of the electromagnetic energy in an open "box" adjacent to the terminating end of the structure of Ref. [2], for both a reciprocal (black dashed–dotted curve; B = 0 T) and a nonreciprocal (red solid curve; B = 0.2 T) structure. For the remaining two curves, please refer to Fig. 3(b) of Ref. [2]. (b) Local electric field in the "box" of the reciprocal structure, recorded at various time instances (in units of $T_p = 1/f_p$, with $f_p$ being the plasma frequency of InSb). (c) Same as in (b), but for the nonreciprocal structure—cf. Fig. 3(a), right panel, of Ref. [2].

the '$1/\tau_{\mathrm{out}}$' *radiative* out-coupling power, not with the total, dissipative '$1/\tau_0$', plus radiative,'$1/\tau_{\mathrm{out}}$', rate). In other words, for Lorentz reciprocity to be broken in a cavity resonator, one only needs to (radiatively) in-couple light energy to the cavity, and then the light energy should not *radiatively* escape the cavity—but all light energy will still, nonradiatively (that is, via heat) "escape" the cavity, as shown in Fig. 3(b) of Ref. [2], and still further herein in Fig. 2. In fact, this is precisely the physical origin of the ∼100% absorption in the whole CUP region reported in Fig. 2 herein. We note that this definition of nonreciprocity in a resonator is completely analogous to the well-known definition of nonreciprocity for a waveguide (also reported as Eq. (2) in Ref. [2]) where the wave *transmission* from a point A to a point B should be different from the wave *transmission* from point B to point A—that is, reference is made to the radiative power, to the transmission (we do not 'send' Joule losses from A to B, or from B to A). Thus, to break Lorentz reciprocity in a resonator too, one needs to make unequal only the *radiative* parts of the in-/out-coupled powers—as was reported and explained in Ref. [2]. The total (radiative + dissipative) in- and out-coupled powers are always equal at steady state, as dictated from Poynting's theorem (which is automatically respected in FDTD simulations).

Further, in Ref. [1] Mann *et al*. observe a localized hotspot, which they refer to as a wedge mode, i.e., the trapped state. They conclude that both their trapped state, as well as the one observed in Ref. [2], are not due to nonreciprocity, but simply an example of plasmonic focusing, i.e., a tapered plasmonic waveguide, with nonreciprocity only providing impedance matching. We will now show that this is a misconception, and that nonreciprocity is fundamental to the performance of the device. Specifically, we will show that in the reciprocal version of the device the electromagnetic energy is not confined to a localized region, and while a field enhancement is observed, it does not represent the same focusing nor enhancement factor. Crucially, we will also show that in the reciprocal case the energy of the trapped state decays in a tiny fraction of that of the nonreciprocal structure—that is, the T-B performance of the nonreciprocal structure is drastically superior.

To this end, Fig. 3 reports FDTD calculations, similar to those presented in Ref. [2], displaying (a) the energy density in the termination as a function of time and (b),(c) the spatial distribution of the electromagnetic energy at various times for both the (b) reciprocal and (c) nonreciprocal cases. In all cases, the same device as in [2] is investigated [cf. Fig. 1(b)], with the difference being that for the reciprocal case no external magnetic field is applied (B = 0 T), while the nonreciprocal case features an applied external magnetic field (B = 0.2 T). Furthermore, the group velocity of the incident light in both cases is almost exactly the same ($v_g = 0.0681c$ for B = 0 T and $v_g = 0.0673c$ for B = 0.2 *T*); thereby any difference(s) in behavior cannot be attributed to conventional slow-light effects. From Fig. 3(a), we clearly see that the reciprocal device has a much faster decay rate than its nonreciprocal counterpart. In both cases, the pulse propagates (slowly, with the aforementioned group velocities) towards the termination. However, once it reaches the termination, the behavior starts to differ dramatically. For the reciprocal structure, the pulse enters the trapping region and is then back-reflected, resulting in a rapid decay of the energy within the open cavity region. For the nonreciprocal case, however, the back-reflection cannot occur (as there is no backwards propagating mode), and light is now trapped in the open cavity region, decaying slowly *only because of dissipative losses* (material absorption). We also note from Fig. 3(a) that if the magnetic field is reversed [−0.2 *T* lines in Fig. 3(a)], then the pulse can be recovered at times much later than for the reciprocal case. Therefore the nonreciprocal structure clearly outperforms the storage capabilities of conventional plasmonic focusing, with the increased storage time, i.e., delay, being a direct consequence of nonreciprocity.

To further demonstrate this argument, we show the electromagnetic field within the trapping region (∼798 μm $< x <$ 800 μm) for both the reciprocal [Fig. 3(b)] and nonreciprocal [Fig. 3(c)] cases, at different times, normalized to the amplitude of the incident pulse. We observe that, for both cases, the maximal field enhancement occurs at a time $t = 100\,T_p$ ($T_p = 1/f_p$, where $f_p$ is the plasma frequency of InSb). However, for the reciprocal case we observe only a ∼5 times amplitude enhancement (i.e., conventional plasmonic focusing), and we see that over the observed spatial region the field is approximately uniform. In contrast, for the nonreciprocal case we observe that at the same time instant the amplitude enhancement is by a factor of $\sim$3300—almost three orders of magnitude above the conventional result (reciprocal structure). Furthermore, the field is confined in a significantly smaller spatial region. The same pattern is observed at all times; i.e., at any point in time the reciprocal structure has a local field amplitude at the focusing tip several orders of magnitude smaller than that in the nonreciprocal structure, and spreads out uniformly in the spatial region of interest, while the nonreciprocal structure displays extraordinary amplitude enhancement and localization of the field in a smaller spatial region. As such, the argument made in Ref. [1] that the observed effect is conventional plasmonic focusing is clearly unfounded and in contradiction with the observed behavior of the device. Both the field enhancement and the T-B performance are dominated by the nonreciprocal nature of the device. These T-B-related differences between nonreciprocal (topological [7,8]) and reciprocal (ordinary) terminated structures become even more pronounced when realistic surface roughness and material imperfection effects are considered, as it is well-known that reciprocal such structures may even lose their ability to focus and localize light at their tip [9], whereas the nonreciprocal structure of Fig. 1(b), being topological [7,8], is completely immune to such effects [7].

Finally, a few points and clarifications are due with regard to the potential role of nonlocality [10,11] on the attained, large, T-B violations, as well as on the nature of the 'open cavity' considered in Ref. [2] and in (the blue spot of) Fig. 1(b) herein. First, the objective of Ref. [2], as well as of the present Comment, was to show that the T-B limit can be exceeded by essentially an arbitrarily high degree in *local* (non-spatially-dispersive), linear, time-invariant structures—that is, the same type of structures considered in Ref. [1], as well as in similar previous works [12–14], which reasoned that no such violations may exist in such structures for fundamental reasons [14]. The results and physical justifications presented here, as well as in Ref. [2], rigorously show that the T-B limit characterizing local, linear, time-invariant structures can be overcome so long as such a violation is *topologically enforced and protected*. Second, even when nonlocal effects are considered, one may always redesign the terminated structure considered here and in Ref. [2], e.g., simply by removing the dielectric (Si) layer, such that it can robustly preserve its unidirectional and topological character even in the presence of nonlocality, and for arbitrarily small levels of dissipation, as has recently been shown in Ref. [11]—thus, nonlocality cannot for fundamental reasons, i.e., for all possible structures, destroy topological protection (topology), since the latter is a deeper and more fundamental property. Third, there is no need for termination and its associated large field enhancement in a tight region [cf. Fig. 3(c) and brief discussion below], which might give rise to nonlocal effects, as ultrabroadband light trapping [15,16] and releasing [17] can also exist in topological (unidirectional) 'trapped rainbow' structures [18,19], which can stretch out and localize (trap) a lightfield in tapered guides in a manner stable even under fabrication disorders [15]. Fourth, for device applications of such T-B violations, other important phenomena, such as nonlinear and thermal effects [19], will need to be considered, both of which can, however, be addressed by, e.g., lowering the injected light power or resorting to cryogenic conditions. It is also to be stressed that the trapped state considered in Ref. [2] and in this Comment is formed by a non-self-sustained [5] bulk (not surface) plasmon [6] of the terminating Ag layer: the $E_x$-field component, perpendicular to the terminating Ag layer, is dramatically enhanced, inducing free charges on its surface (bulk plasmon), and it is the near field of that bulk plasmon that the pulse is in-coupled to, without reflection(s) across the entire CUP region. Such plasmonic particles, and their associated bulk plasmons, are typically referred to as 'open cavities' in the field of (nano)plasmonics [6]. Therefore, the lossy topological [7] 'open cavity' (i.e., the Ag particle) in Ref. [2] and herein is fundamentally different from the lossless perfect-electric-conductor ordinary cavity terminating the unidirectional waveguide of Ref. [1], which, therefore, not surprisingly, does not reproduce the behavior reported in Ref. [2].

In conclusion, the paper by Mann *et al*. [1] makes an interesting contribution in that it convincingly shows that any system whose dynamics are accurately described by (the standard, single-resonance form of) a TCMT approximation is T-B limited, even when nonreciprocally fed. However, by not recognizing the afore-outlined inherent limitations of such a method of analysis, and the fundamental differences of the structure they considered compared with that in Ref. [2], Ref. [1] reached the generalized conclusion that all (local) linear, time-invariant structures are T-B limited, including the one shown in Fig. 1(b) herein, studied previously in Ref. [2]—a conclusion that is unwarranted, as explained in some detail above. Moreover, the assertion of Ref. [1] that nonreciprocity does not beget any specific advantage(s) in terms of the T-B performance of a device is unjustified too, as was clearly shown in Fig. 3 above. Thus, overall, this Comment helps to clarify that the time-bandwidth limit *can* be exceeded, in fact to an arbitrarily high degree as Ref. [2] has previously reported, even in (local) linear, time-invariant structures, that topology and nonreciprocity play a crucial role in achieving this feat, and that the standard form of, otherwise powerful, quasi-analytic techniques, such as TCMT that was deployed in Ref. [1], fails to accurately describe the dynamics and physics of (open) nonreciprocal cavities, even in the low-loss regime where they are normally successfully applied.

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† These authors contributed equally to this work.

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