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ABSTRACT

Topologically protected transport has recently emerged as an effective means to address a recurring problem hampering the field of slow light for the past two decades: its keen sensitivity to disorders and structural imperfections. With it, there has been renewed interest in efforts to overcome the delay-time-bandwidth limitation usually characterizing slow-light devices, on occasion thought to be a fundamental limit. What exactly is this limit, and what does it imply? Can it be overcome? If yes, how could topological slow light help, and in what systems? What applications might be expected by overcoming the limit? Our Perspective here attempts addressing these and other related questions while pointing to important new functionalities both for classical and quantum devices that overcoming the limit can enable.

INTRODUCTION

Resonances and resonant cavity devices are ubiquitous in wave physics and engineering—from nanophotonics, metamaterials, and Si photonics to atomic and molecular optical physics and condensed matter.[1–7] Well-known types of resonances range from standard resonances, such as Lorentzian, Fano, Mie, Raman, and so forth, to more exotic ones such as embedded eigenstates in the continuum or parity-time-symmetric resonances. Devices based on resonances (resonators) are pervasive components of virtually all mainstream modern technologies, from lasers, sensors, filters, and antennas to modulators, detectors, spectrometers, and integrated electronic or photonic circuits. In the nascent field of "metamaterials," for instance, the very first such a (meta)material was an array of split-ring *resonators*.[2,8]

Despite this, quite literally the totality of present-day resonant devices and cavities are well-understood to be time-bandwidth (T-B) limited[9–13]—that is, *for a given (fixed) footprint*, the wave-storage time Δt of any such devices, be it resonant or waveguiding, is inversely proportional to the device's bandwidth, $\Delta\omega$, with the insurmountable product of the two usually referred to as the T-B limit[9,10] (see Fig. 1 and the section entitled as "What is the time-bandwidth limit?"). This inverse proportionality relation between Δt and $\Delta\omega$ has adverse consequences for the performance of all such devices: We may spend years, or even decades,[14,15] to optimize their wave-storage (low loss) performance, i.e., increase "Δt," typically in order to enhance wave–mater interactions for a sought-after functionality, but that automatically leads—always—to a correspondingly narrower device bandwidth $\Delta\omega$—an undesirable characteristic for a range of key applications requiring the broadband passive performance, particularly in communication systems and integrated nanoelectronics and nanophotonics. Furthermore, the narrow-bandwidth characteristics of low-loss resonant devices directly imply that these (passive) systems are correspondingly slow, too, since the higher the Q-factor of a system (i.e., the narrower the bandwidth) the longer it takes to respond to the external stimuli[2,9]—making them unsuitable for applications requiring "ultrafast" device-accessing (in-coupling) and -exiting (outcoupling) time scales.

This "fundamental" T-B limit constrains not only resonant devices but also waveguiding devices of *fixed* length[9–13]—e.g., it is perhaps the most well-known limitation of present-day "slow light" schemes. Thus, virtually all devices, throughout wave physics and engineering, be them resonant or waveguiding, are T-B limited—since any device *of fixed footprint* either "stores" a wave for some time, i.e., it functions as a cavity or propagates a wave over some distance, i.e., it functions as a waveguide. Furthermore, cutting-edge systems, such as light-(nano)focusing nanoscopes,[17] and chiral sensing schemes[18–20] also fall victims of this limit. Indeed, as explained in more detail in Refs. 17 and 21, the T-B

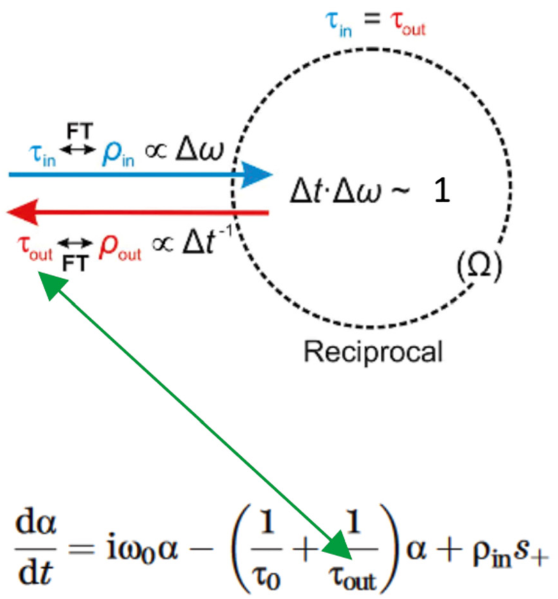

FIG. 1. Physics and time-bandwidth characteristics of reciprocal and open nonreciprocal cavities. In a standard reciprocal cavity (upper left panel), the in- and outcoupling *radiative* wave-energy rates (ρ_{in} and ρ_{out}) are equal, whereas in an open nonreciprocal cavity, it is $\rho_{in} \gg \rho_{out}$ over a broad band $\Delta\omega$ for an arbitrary incident pulse. Because $\rho_{out} \to 0$, the pulse stays trapped (localized) in the open nonreciprocal cavity for long times Δt, giving rise to $\Delta\omega\Delta t$ products that can exceed all known forms of the time-bandwidth limit. The bottom row shows the standard single-resonance temporal coupled-mode theory equation,[3,16] describing accurately only reciprocal (ordinary) cavities, where τ_{out} is the *radiative*-loss lifetime of a wave of amplitude α inside the cavity, and s_+ is the incident power. Adapted from Ref. 9.

limitation is responsible for the very low throughput efficiencies and *low speeds* of, e.g., near-field scanning optical microscopes (NSOMs)—the incumbent technology for bringing light to the nanoscale—and for similar limitations in a whole range of key nanoscale technologies.[21] Likewise, the sensitivity limits of commercially available optical polarimeters, circular dichroism, and optical rotation modules are currently limited by the insufficiently high intra-cavity fields and light powers, corresponding to analyte concentration detection limits at the (sub)-micromolar levels, thereby constraining the extension of polarimetry to a wide range of important research and industrial applications requiring improved sensitivity levels (e.g., sub-nM levels).[18–20]

With these general remarks in mind, in the following three and in the last two sections, we outline in more detail how exactly this limit arises, why overcoming it with topological slow/stopped light and nonreciprocal cavities is, both, feasible and important, the meaning of "nonreciprocal cavities" within the present context, and the new functionalities and applications that could be enabled by overcoming the limit.

WHAT IS THE TIME-BANDWIDTH LIMIT?

Consider a resonant device storing (localizing) a wave of amplitude W inside it. The wave oscillates sinusoidally, say, with a frequency ω_0 and decays with time owing to some loss mechanism(s) with a total decay rate[9,10,21,22] Γ, i.e., $W(t) \propto \cos(\omega_0 t) \times e^{-(1/2)\Gamma t}$. Then, in the resonance approximation and in the usual underdamped regime $(\Gamma/2 \ll \omega_0)$, the energy spectral density $E(\omega) = |\mathcal{F}\{\langle \dot{W} \rangle\}|^2$ of $\langle \dot{W}(t) \rangle$, where $\langle \cdot \rangle$ indicates the operator extracting the envelope of its arguments and $\mathcal{F}$ is the Fourier-transform operator, will be given by

$$E(\omega) = \frac{1}{(\Gamma/2)^2 + \omega^2}. \quad (1)$$

From the above relation, it can be seen that the (half-amplitude) bandwidth of the resonant device is $\Delta\omega = \Gamma$. In other words, the product of the device's storage time, $\Delta t = 1/\Gamma$, with $\Delta\omega$, appears to always, for any linear time-invariant (LTI) system, be equal to unity, $\Delta t \Delta\omega = 1$—a limit known as the "time-bandwidth (T-B) limit" of resonant devices.

This limit is a completely general phenomenon, characterizing the storage capacity of all linear, time-invariant, resonant, and wave-guiding alike, devices—from photonics to acoustics, opto-mechanics, atomic and molecular physics, as well as mechanical and structural systems.[9–13] It should not be confused with the mathematical time-bandwidth limit, $\sigma_t^2\sigma_\Omega^2 \geq 1/4$, where σ_t^2 is the time variance of a signal $x(t) \in L^2(\Re)$ and σ_Ω^2 its frequency variance, i.e., with the uncertainty principle characterizing Fourier-integral pairs in signal analysis and communication systems and which, among others, only has a lower bound. Although both limits often bear the same name, the T-B limit discussed here (which has an upper bound[9–13]) characterizes the storage capacity of the devices themselves—not the mathematical Fourier properties of the respective signals.

In addition to resonant devices, the physical T-B limit discussed here also arises, albeit in a different form,[9–13] in guiding structures *of fixed length*, such as slow-light waveguides or bulk media (of fixed wave-propagation length). Here, because there is propagation (unlike resonant cavities), the role of group-velocity dispersion and attenuation also need to be considered, and a number of works have elucidated that a structure can decelerate a wave over only a finite bandwidth $\Delta\omega$ inversely proportional to the group index n_g. Hence, a structure *of fixed length L* cannot delay a wavepacket of bandwidth larger than $\Delta\omega$ by more than a time $\Delta t \sim n_g L/c$, where c is the speed of light in vacuum. In other words, the "delay-bandwidth product," $\Delta t\Delta\omega$, characterizing waveguiding structures has an upper limit. Intuitively, this arises because in order to increase the delay Δt in a guiding structure of a given (fixed) length, the group refractive index $v_g = \mathrm{d}\omega/\mathrm{d}\beta$, where β is the longitudinal propagation constant, needs to decrease, i.e., the dispersion band needs to become "flatter"; therefore, the bandwidth $\Delta\omega$ over which v_g is defined has to decrease. Careful analyses, incorporating the role of group-velocity dispersion and attenuation, show that, in fact, in this case, for waveguides, Δt becomes inversely proportional to a power[9–13] of $\Delta\omega$—e.g., $\Delta t \sim \Delta\omega^{-\alpha}$, $\alpha = 2, 3$; an even stricter limitation.

In all cases, the T-B limit simply states that it is impossible to store, "trap," or buffer *broadband* waves for *long times* in any known LTI devices of fixed size, throughout wave physics and engineering—a major limitation, often argued to be fundamentally insurmountable. One may also note here that unlike dissipative losses in plasmonic structures, which are often thought to constitute a key issue in that broad field, the T-B limit also concerns dielectric structures as well, i.e., it is a broader and apparently more severe limitation.

WHY OVERCOMING THIS LIMIT IS IMPORTANT?

There are two main reasons for this. First, for waveguiding structures of fixed length, the product $\Delta t\Delta\omega$, being a unitless number, gives the number of "bits" (wave pulses) that can be stored or buffered in that structure, i.e., it is a direct figure-of-merit of the "capacity" or buffering "memory" of that system[9–13]—the larger the product $\Delta t\Delta\omega$ becomes, the more bits of information we can temporarily be stored (buffered) in the structure. It should here also be noted that one of the key advantages of "slow light" is the fact that it allows for shorter device length for a given functionality. Indeed, as it is well-known (e.g., Refs. 3, 9, 11, and 12), a given change δn in the material refractive index required for a specific functionality (e.g., in an interferometer) corresponds to a shift in an electromagnetic band by a fixed amount $\mathrm{d}\omega$, because the frequency is kept constant by the choice of the operating wavelength. As a result, a larger group refractive index $n_g = c/v_g = c/(\mathrm{d}\omega/\mathrm{d}\beta)$, that is, a "slower" wave, requires a shorter device length $L_\pi = \pi/\mathrm{d}\beta$ for a required phase shift (here, π). Hence, "slow" light is usually exploited for reducing a device's footprint.

Second, and perhaps more importantly for micro- and nano-photonic applications, as has recently been shown,[23] the product $\Delta t\Delta\omega$ is also a direct figure-of-merit of the *enhancement* (compared to a T-B limited device) in wave *power* that a resonator or, waveguide of fixed length, can accept—i.e., a large value of $\Delta t\Delta\omega$ leads to a correspondingly-higher wave power inside the resonant or waveguiding system. This is particularly important for a host of applications relying on strong wave-matter interactions, including efficient high-harmonic generation, enhanced and faster (broadband) spontaneous emission rates, sensing, strong wave-matter coupling, single- or few-photon nonlinearities, and so forth. More examples of this are discussed later on, in the last two sections, in this Perspective.

WHAT IN THE PRESENT CONTEXT IS MEANT BY A "NONRECIPROCAL OPEN CAVITY"?

A nonreciprocal cavity is a cavity for which, at steady state, the rate of electromagnetic energy in-coupled to the cavity is larger, *over a broad range of frequencies*, than the rate with which energy *radiatively* out-couples from the cavity (cf. Figs. 1 and 2). In other words, without the energy building up in the cavity (steady state), there is more wave-energy injected into the cavity than wave-energy radiatively escaping the cavity. This can, e.g., happen when a wave pulse enters a finite

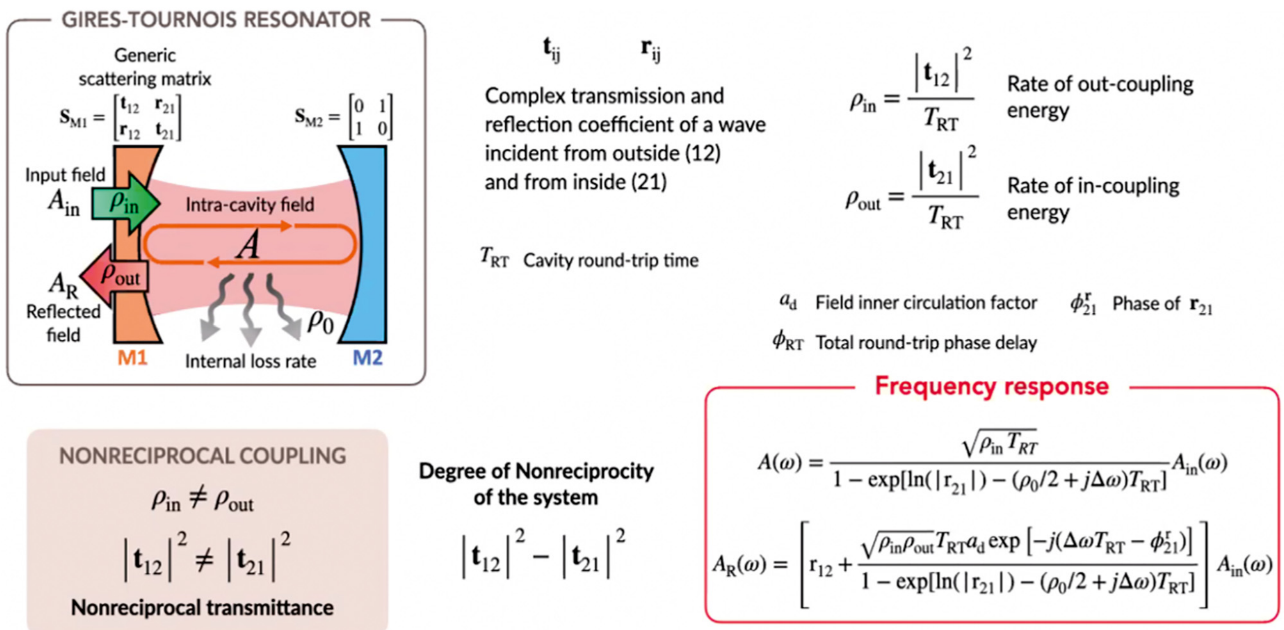

FIG. 2. General schematic of a nonreciprocal cavity along with the main definitions used for the analysis of its time-bandwidth product and intra-cavity power.

region of space (an open "hotspot" or an open cavity[24]) and is therein completely trapped without the lightfield escaping the region radiatively. Such a configuration is often referred to as an "open" cavity,[7,24] i.e., a cavity that does not feature ("hard") material boundaries, and inside which a pulse can enter even seamlessly with minimal back-reflections (high in-coupling energy rate).

DOES THE INEQUALITY OF RATES VIOLATE ENERGY EQUILIBRIUM?

The inequality of in-coupling and outcoupling energy rates over a broad band in a nonreciprocal cavity might at first sight seem to violate energy balance at equilibrium (steady state). Poynting's theorem stipulates that at the steady state the rate of energy coupled into a given (finite) region of space must be exactly equal to the *total* (radiative + dissipative) rate of energy out-coupled from that region (cavity). However, it is here to be pointed out that this equality concerns the total rates: If all the wave energy couples to the cavity (large in-coupling rate ρ_{in}) and no wave energy *radiatively* out-couples from the cavity (zero *radiative* outcoupling rate, $\rho_{\text{out}} \to 0$), then necessarily all the in-coupled and trapped wave energy will be dissipated in that finite spatial region, i.e., the *total* in-coupling energy rate will still be exactly equal to the *total* outcoupling rate (radiative + dissipative $= 0 +$ dissipative $=$ dissipative only in this example).

Such a cavity, where all the wave energy couples inside it, without reflection, and is eventually dissipated in the cavity, essentially behaves as a perfect absorber—sometimes also termed, more exotically, as an "optical black hole"[25] because light cannot radiatively escape from it. Note that the inequality in the radiative-only part of the rates is required, because we are solely interested in decelerating the field itself—not the onset of heat. Thus, if, for instance, $\rho_{\text{in}}^{\text{rad}} \sim 1$ and $\rho_{\text{out}}^{\text{rad}} \sim 0$, then a pulse seamlessly enters a given finite region and is therein be trapped (not exiting *radiatively* that region, $\rho_{\text{out}}^{\text{rad}} \sim 0$), thereby increasing Δt (the storage or delay time) over the whole bandwidth $\Delta\omega$ of the open device (the open resonator or the waveguide of fixed length). If the region is low-loss, then the characteristic delay time Δt required until the pulse is completely dissipated (if it does not exit the device at all) will be very large—and decoupled from $\Delta\omega$. Consequently, T-B limits of any form[9–13] can in this manner be exceeded, since the attained delay (storage time) Δt may, in principle, become much larger than $\Delta\omega^{-1}$ (determined solely by the device's material losses, not by an inverse proportionality to $\Delta\omega$). Note that the overall losses could also be controlled by, for instance, judiciously introducing a "scattering" channel (e.g., to a different port); therefore, this may be a potential way for enabling and *controlling* very high intracity power inside open nonreciprocal cavities—up to the onset of nonlinear (if not needed) or material-damage effects.

The ultimate limit on how large the T-B product can become is set by the dissipative rate in the open nonreciprocal cavity, that is, from its "finesse," F. In recent works,[23,47] it has been shown that this ultimate limit on the T-B product (TBP) is given by the following relation:

$$\text{TBP} = F/(2\pi) + 1, \tag{2}$$

a relation which holds true, both, for LTI and time-varying structures—the only difference being in the precise mechanism via which the in-coupling loading of the cavity is in each case realized. This relation implies that not only can the standard TBP (= 1) be overcome but that, in fact, it can attain huge values of the order of the finesse of a cavity, that is, many orders of magnitude in suitably designed extremely low-loss open structures—orders of magnitude above the standard "unity" (= 1) T-B limit of closed ordinary cavities.

WHAT IS THE DIFFERENCE WITH "CRITICAL COUPLING" IN CONVENTIONAL CAVITIES?

Critical coupling in standard cavities[1,3,14] refers to the situation where all the incident wave energy in-couples to the cavity, e.g., from a waveguide, without any reflection—thus, at first sight, this standard scenario might look similar to the one described above for nonreciprocal cavities, since in both cases light is in-couped to a system and there is no radiative (optical) outcoupling. However, there is a key difference, directly affecting how the T-B limit can be overcome: In nonreciprocal cavities, the reflectionless in-coupling occurs *over broad bandwidths* even for very low-loss ($\Gamma =$ small) structures, for which the bandwidth is otherwise (in standard cavities) narrow ($\Delta\omega = \Gamma$). The difference in the bandwidth performance between nonreciprocal open cavities and critical coupling in conventional (reciprocal) closed cavities can, thus, be very large, as Fig. 3 shows—with this enhanced bandwidth performance leading ultimately to overcoming the T-B limit by a large degree.[9,10]

PHYISCALLY, HOW IS THE TIME-BANDWIDTH LIMIT OVERCOME IN NONRECIPROCAL OPEN CAVITES?

In order to delay a wave, what is required is a mechanism for decelerating or altogether preventing its propagation, "holding" it in a given finite region of space. One way of doing this is via resonant cavities where instead of propagating, a wave accumulates in the cavity, building up its amplitude from the incident wave energy. Conventionally, for this to happen, what is required is a 2π phase shift over a round trip inside the cavity (for a given wave-frequency ω_0), so that constructive interference over multiple roundtrips leads to the energy accumulating inside the cavity. For standard cavities, this

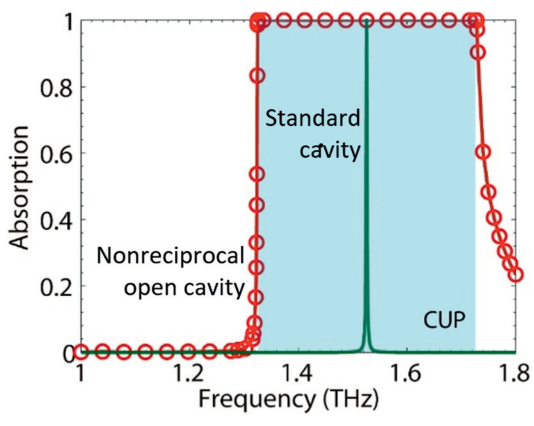

FIG. 3. Absorption resonance in a conventional cavity (green line) and in a nonreciprocal open cavity (red line with symbols) with the same loss $\gamma_i \sim 3.0614 \times 10^9$ rad/s [cf. Eq. (2)]. In this example, the nonreciprocal open cavity (hotspot) is formed at the end of a terminated unidirectional waveguide, operating in its complete unidirectional propagation (CUP) region. Adapted from Ref. 10.

process is mathematically described by the following equation[3,9,10,16] (see also Fig. 1):

$$\frac{da}{dt} = i\omega_0 a - (\gamma_i + \gamma_r)a + \kappa_{\text{in}} s_+, \tag{3}$$

where a is the field amplitude inside the cavity, ω_0 is the resonance frequency of the (assumed, single) cavity mode, γ_i and γ_r are the intrinsic and radiative loss rates, respectively, $|s_+|^2$ is the power incident onto the cavity from an external system, e.g., a waveguide, and κ_{in} is the coupling coefficient between that external system and the cavity. Here, the key term is "i$\omega_0\alpha$," where ω_0 is the peak of the resonance in the cavity.

The open resonance in a nonreciprocal cavity, on the other hand, is a broad, flat-top (no single-peak/plateaued) *open* resonance (a resonance of an open resonator) with no well-defined single ω_0 peak (on a linear scale), as illustrated in Fig. 3. Physically, this flat-top open resonance can be attained within the unidirectional band of a terminated topological waveguide:[9,10] Because the terminated guide allows propagation strictly in only one direction, forbidding back-reflections from its end, or other scattering, a *broadband* pulse can rigorously be trapped at its end in a topologically enforced and protected way[10,26] without requiring a "hard" boundary at its trailing end as in conventional cavities—that is, without giving rise to a standing wave ["i$\omega_0\alpha$" term in Eq. (2)]. In this way, broadband radiation can be slowed down and trapped for times much larger than $\Delta\omega^{-1}$, the inverse bandwidth of the structure, breaking the T-B limit by a large degree. In Fig. 3, in particular, the degree to which the limit is overcome is precisely the degree to which the bandwidth of the nonreciprocal cavity (red line) is broader than the bandwidth of the conventional cavity (green line) of the same intrinsic loss—in this case, by a factor of $\sim$1000.

CANNOT THE SAME BROADBAND TRAPPING OCCUR IN TAPERED, PLASMONIC OR OTHER, WAVEGUIDES?—ROLE OF TOPOLOGY

In the ideal macroscopic theory, a guided pulse could, in principle, be stopped (zero group velocity, $v_g = 0$) and localized in tapered plasmonic or other waveguiding structures.[21] However, it is by now well appreciated that such a state is not stable in the optogeometric parameters space, that is, small, unavoidable (nm-scale, or less) surface roughnesses, defects and material inhomogeneities prohibit the stopping and localization. Physically, this occurs because a zero group-velocity corresponds to an infinite group refractive index, which practically cannot be attained. This is because in the stopped-light regime, a guided wave has even more time to interact with the aforementioned structural imperfections, as a result of which it is eventually either back-reflected[27–29] or even split in parts and scattered.[30]

Thus, nonreciprocity and topology[31]—both, important even in the usual light regime of integrated optoelectronic and photonic structures—are particularly crucial in the slow- and stopped-light regime and, as a number of recent works have elucidated,[32–44] play a key role for attaining prolonged *and* broadband (i.e., T-B unlimited) light localization in topological, terminated, or "rainbow trapping" structures. In the latter approach, use can be made of the concept of synthetic dimensions.

A synthetic dimension in graded rainbow trapping structures can be constructed by exploiting a translational degree of freedom[34]—e.g., inside the unit cell of a two-dimensional chirped photonic crystal waveguide. The translational grading (tapering) gives rise to a nontrivial topology in the synthetic dimension, which, in turn, results in robustly localized (stopped) surface states where different frequencies are localized at different positions along the guide ("rainbow

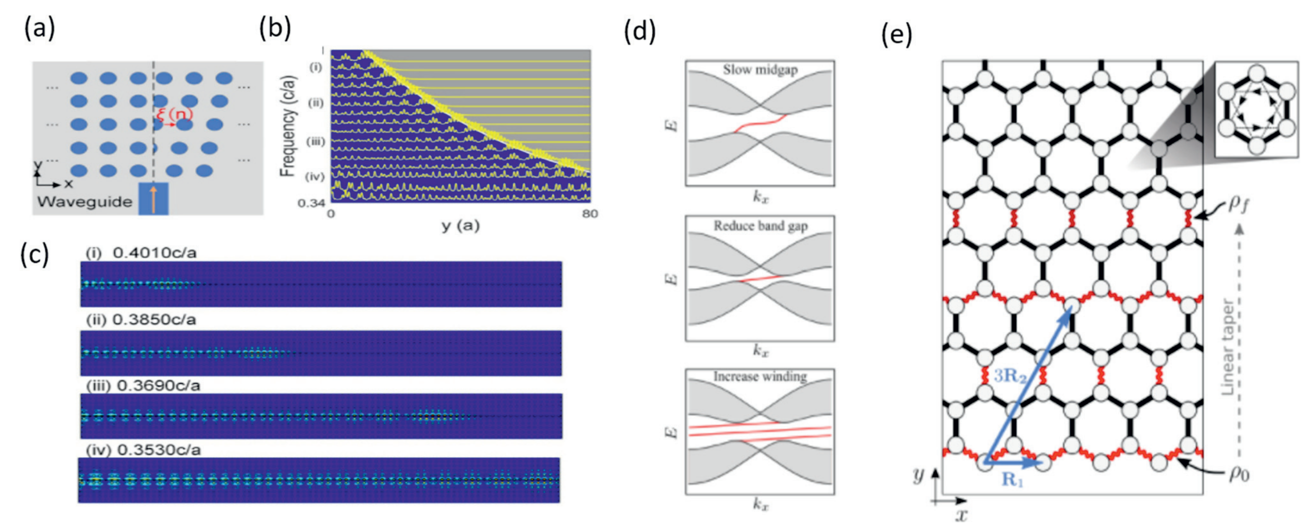

FIG. 4. (a) Schematic diagram of a topological "trapped rainbow" structure with light coupled-in using a dielectric waveguide. (b) Normalized energy density distributions along the interface shown in (a). The blue and gray parts are regions of existence and nonexistence of interface modes. (c) Different light frequencies being rigorously stopped and trapped at different positions along the topological guide of (a). (d) Comparison between the band structures of different methods for generating slow topological edge modes. The first two methods illustrate how, conventionally, slow edge modes exist over a narrow range of frequencies, while the third (lower) method gives rise to a slow edge mode with a large bandwidth. (e) A method for generating the band diagram of the lower panel in (d), illustrating the edge termination with wavy red lines indicating reduced nearest-neighbour couplings. The couplings are reduced by a factor that begins with ρ_0 at the edge and *linearly tapers* to a final value of ρ_f. Up to these factors, the nearest-neighbour coupling pattern repeats n times (here, $n = 2$) under translation by 3$\mathbf{R}_2$ before terminating at the bulk of the structure. Adapted from Refs. 32 and 35.

principle,"[21] see Fig. 4). Such "topological trapped rainbows" can rigorously stop and localize states of different frequencies at different positions along a topological waveguide, controlled by the tuning of the spatial modulation of the states' group velocity.[27,32,35,40–44] For photonic crystal structures, in particular, the operation frequency as well as the bandwidth of the topological trapped rainbow can be tuned by controlling the bandgap of the photonic crystal and is completely decoupled from the storage (stopping) time.[34] This topological principle can be applied to photonic crystals of any symmetry and material composition, so long as a complete bandgap exists.

A further topological approach for attaining *broadband* topological slow is based on engineering the edge termination of a periodic structure, by which a topological edge mode can be made to wind many times around the Brillouin zone as it crosses the bandgap, thereby giving rise to slow light *over a large range of frequencies*.[32] The number of times the edge mode winds around the zone is determined by the depth of the linearly tapered modification (rainbow principle,[21] see Fig. 4) of the edge termination perpendicular to the direction of propagation. In the direction parallel to the edge, the termination does not expand the size of the unit cell; therefore, it generates multiple windings differently from simple band folding. Note that since here light is slowed down without reducing the width of the bandgap, its existence remains protected against even strong disorder, i.e., as long as disorder does not close the large topological bandgap. With this approach, too, one is able to overcome the mechanism by which the T-B limit usually arises in waveguides: Normally, in order to slow down light and increase the delay Δt, one has to decrease the group velocity $v_g = d\omega/d\beta$, but that automatically reduces the bandwidth $\Delta\omega$ over which Δt increases, thereby giving rise to the standard limit (for an assumed fixed length of the guide). Here, however, this mechanism is not invoked, as the ability to slow down the mode *without reducing its bandwidth* is enabled by the fundamentally 2D nature of the system with different frequencies residing at different depths in the structure ("rainbow" principle). As a result, the minimal group velocity (maximal Δt) attainable at a fixed bandwidth $\Delta\omega$ (and a fixed device length L) is determined by the system size in the direction *orthogonal* to the direction of propagation, not by an inverse proportionality to $\Delta\omega$—i.e., Δt and $\Delta\omega$ are decoupled, which is the very definition of overcoming the T-B limit.

In terminated topological structures,[9,10] on the other hand, wave propagation is halted by the use of a terminating metallic layer (not by reducing v_g to zero), where back-reflections and scattering are prevented owing to the topological design of the structure. This form of prolonged localization too survives from all of the aforementioned realistic material effects, as well as from dissipative losses and nonlocality when suitably designed. Both classes of structures, as well as (a third approach) the use of multiple resonances, judiciously spaced spectrally, on metasurfaces whereby one can combine the strong delay of constituent resonances with the broad aggregate bandwidth of the resonances ensemble while ensuring spectrally constant aggregate bandwidth—an approach which has now been demonstrated, both, theoretically and experimentally[45,46]—form a mounting body of works[9,10,21–47] showing that even linear time-invariant (LTI) devices can slow down or even localize and trap light pulses beyond the T-B limit of standard resonant or waveguiding structures. We note, in passing, that in the case of terminated waveguides it is not strictly necessary to reduce the group velocity to "zero," since light is therein trapped regardless of its group velocity; in this case, therefore, the material-sensitive zero-v_g point (divergent group refractive index) is avoided, and "slow" light has to be understood in terms of the time delay that the trapped light still experiences, i.e., it is slow (time delayed) in that sense strictly.

It also to be noted that, in general, topological corner states do not break the T-B limit, because these states are not, simultaneously, nonreciprocal—they arise at the corners of a higher-order (photonic) topological insulator (HOTI) and a topologically-trivial material, but the band structure of the HOTI is usually symmetrical around $k = 0$, i.e., reciprocal. As Fig. 2 shows, in order to break the T-B limit, one needs to make the in-coupling energy rate ρ_{in} in an open cavity be different (larger than) the outcoupling energy rate ρ_{out} ($\rho_{in} \gg \rho_{out}$), so that an injected pulse stays inside the cavity (i.e., is delayed), without exiting it, for long times ("Δt" large). This ($\rho_{in} \neq \rho_{out}$) can only occur if the structure, in addition to being topological (for providing robustness to material imperfections and nonlocality), is also nonreciprocal. However, if such HOTI corner states are designed such that they also feature broken time-reversal symmetry (i.e., be nonreciprocal/asymmetric around $k = 0$), then these states too could be used for overcoming the T-B limit.

WHAT APPLICATIONS COULD BE ENABLED BY OVERCOMING THE LIMIT?

As has recently been established,[23] the T-B product of a device can also be seen as a practical figure-of-merit for the enhancement in intra-cavity power—the degree to which the limit can be exceeded exactly corresponds to the degree to which light power increases inside a device. Thus, devices operating beyond the limit will be uniquely positioned to enhance optical nonlinearities or achieve ultrafast active control.[49–57] Nonlinear photonic interactions (e.g., cross-phase modulation, self-phase modulation, and so forth) are characterized by the various nonlinear susceptibilities of a medium, and such materials enable a host of application-rich effects. The nonlinear interactions can here be greatly enhanced due to the compression of the local energy density as a *broadband* pulse is decelerated inside the device.[21] This could allow for nonlinear processes to occur at much lower operating powers than conventionally required, potentially leading to improved optical switching, additional signal processing capability, and low-power wavelength conversion.

With the above feats in mind, a further key property of active such devices will be their ability to decelerate *multiple* (rather than only one, as in the case of a T-B limit ~1) light pulses and apply on them adjustable optical-signal delays. Existing optical-delay techniques, such as fiber delay lines, Bragg gratings, or free-space optics, lack delay control and suffer from the limited tuning range. The emergence of optical delay lines with enhanced tunability, lying at the heart of many system-level optical signal processing devices and techniques, could enable applications such as tunable synchronizers, switched data buffers, and ultra-compact filters.

The enhancement of nonlinearities can be large enough to produce measurable effects due to single-photon input fields—an effect already well-established even for T-B limited slow-light devices. There is a principal way a single-photon nonlinearity could here be useful as a source of single (e.g., Foch-state) photons with controllable spatio-temporal characteristics.

Indeed, the controlled and consistent production of single photons is an essential enabling technology for the field of quantum information science. At present, non-classical (i.e., sub-Poissonian) single photons are generated by conditional acceptance of photon pairs produced via spontaneous downconversion processes in nonlinear crystals. While this is a vigorous approach, there are key limitations to the use of nonlinear crystals. Indeed, the characteristic correlation time of the produced photon pair is typically on the order of a picosecond, which implies that the use of correlated photon pairs must have transmission distances hundred(s) of micrometers in order to be measured via, e.g., Franson-type interferometry—a challenging task as the propagation length increases. Furthermore, non-classical photons with large bandwidths pose detection and measurement challenges too—detectors need to be broadband, leading to more noise within a detection channel, while generating single-Foch states via transmission of one, conditional on the detection of the other, becomes challenging as well. Broadband slow light with the afore-described characteristics could lead to the production of correlated photons at rapid rates with coherence times orders-of-magnitude enhanced and potentially ease similar integration problems occurring when sending non-classical photons over large distances.

Further envisaged applications of beyond-the-limit slow and stopped light, the regime, where the density of states and the Purcell factor dramatically increase, would be ultralow-light-level all-optical switching, enhanced chiral nanobiosensing, optical micro-combs, and enhanced spontaneous emission rates—appealing for fast light-emitting diodes (nano-LEDs) with an ultimate goal of attaining spontaneous emission rates faster than around 50 GHz, so that LEDs could become faster than lasers.[54–56] All devices relying on strong light-matter interactions[48] would benefit from the above regime because, as explained before, the degree to which the T-B limit is exceeded exactly corresponds to the degree to which power inside a device increases, thereby potentially enabling unprecedented capabilities in the fields of metamaterials, plasmonics, nanophotonics, and nonlinear optics—both, classical and quantum.

OUTLOOK

The objective of this Perspective has been to raise awareness about the important and wide-ranging limitations arising from the time-bandwidth limit and that a growing body of recent works[9,10,21–47] point to the fact that this limit can be overcome—even in linear time-invariant structures. To be clear, as has been established,[11–13] this is indeed a real limit, in the sense that, as explained above, for a given (fixed) device length or footprint, it does currently severely restrict the performance of many contemporary devices—but the aforementioned body of works makes one confident that the limit is not fundamental and should rather be seen similarly, e.g., to the diffraction limit, which—it too—is a real limit, but can be—and has been[2]—overcome with various techniques. We believe that if losses in the fields of metamaterials and plasmonics are an important limiting factor, to which a great of effort has been devoted,[56,57] similar attention should be directed toward overcoming the T-B limit, which has even broader implications and is not restricted only to metallic structures but concerns dielectric devices too.

Three methods have been reviewed for overcoming the limit in LTI structures—topological rainbow trapping,[27,32,35,40–44] electric or magnetic termination of unidirectional or topological waveguides,[9,10] and multiple (spectrally interleaved) sharp resonances on metasurfaces.[45,46] The first method allows for rigorous, topologically protected, broadband stopping of waves in adiabatically graded (chirped, axially varying) waveguides with each frequency of a pulse being trapped at different spatial locations (spatial demultiplexing). Being topological, the method is immune to such deleterious effects as material losses and/or dispersion, structural imperfections, and nonlocality. On the other hand, if metamaterial implementations are pursued, which involve more intricate fabrication, the scheme may be more suitable for larger wavelength regimes, including acoustic, elastic, seismic, and mechanical or thermal waves.

The second method makes use of nonreciprocal, abruptly-terminated waveguides, which allow for trapping broadband guided pulses for long times—much longer (e.g., by a factor of ~1000) than the inverse of the bandwidth of the device, i.e., greatly above the T-B limit. These structures, too, similarly to their topological rainbow trapping counterparts, are insusceptible to structural imperfections and material losses or dispersion but may be affected by nonlocality—although in practical (lossy and dispersive) devices, the role of nonlocality in opening a backreflection escaping channel is, realistically, minimal. Such a channel can be altogether eliminated by making these structures topological, e.g., simply by removing a dielectric material layer and by exciting an upper dispersion band. Furthermore, being nonreciprocal, that is, rigorously allowing for one-way propagation, these structures need not be adiabatically tapered, which can greatly reduce their length and lead to more compact device footprints. Moreover, being abruptly terminated and preventing back-reflections, the same structures allow for the generation of broad-bandwidth, extremely intense hotspots with electric-field enhancements of the order of 10^4 (some of the highest ever reported),[10] ideal for boosting nonlinear and light-matter interaction effects. It is also to be noted that in these structures, the group velocity of the guided pulse need *not* be reduced to zero, since trapping is achieved in an open cavity exploiting an impenetrable barrier and nonreciprocity to avoid backreflection, thereby a potentially "structurally sensitive" divergent group refractive index is bypassed with rigorous trapping being attained regardless of the group velocity of the guided pulse.

The final, third, type of LTI structure for overcoming the T-B limit, for which there is, both, theoretical and experimental evidence on its efficacy, but which nonetheless is not the main focus of the present Perspective, exploits closely spaced multiple resonances, and its main characteristics are outlined herein for the sake of completeness. The method exploits ultrathin, deep-subwavelength, achromatic metasurfaces, in which a spectrally constant group delay is ensured across the combined bandwidth of the resonances, thereby being able to delay arbitrary-broadband pulses (depending on the number of closely-spaced resonances used) without group-velocity distortion. This method is well-suited for the diverse applications that metasurfaces have found, and an essential next step would be to extend it to optical wavelengths, where metalenses and optical holograms are key envisaged devices. Also interesting would be to explore the degree to which the T-B limit can practically (in the presence of structural imperfections) be exceeded in these structures with first experiments showing already a promising potential.

We note in passing that there are also experimental works reported making use of breaking either the "L" (linearity)[23] or "T" (time invariance)[47] assumption to bypass the limit. Since the fields of topological photonics and acoustics[31,58] are currently very active, and growing, one can be hopeful that experimental demonstrations of the first two methods, too, will soon emerge. Theoretical works reporting time-varying schemes for bypassing the limit have also been reported,[59] and those schemes too can trap broadband waves—though it is not yet clear how resilient they potentially are to perturbations and material or structural imperfections.

We also note that most theoretical studies to date have focused on two-dimensional structures, but nonetheless the extension to three dimensions is quite straightforward and has been reported in the literature in several studies.[60] For instance, YIG-based two-dimensional unidirectional waveguides, cladded with perfect electric conductor (PEC) metals on the upper and lower *xz* planes of the guide, give rise to transverse electric (TE) modes, where (if *x* is the direction of propagation) the three components of a mode are E_z, H_x, and H_y. Here, the *z*-direction electric-field component makes it possible to cover the structure with a PEC metal on two *xy* planes too, thereby making the structure three dimensional with no influence at all on its one-way properties, because the electric-field component remains perpendicular to the PEC boundaries on the *xy* planes.

On the other hand, it is interesting to note that when applied to periodic *photonic crystal* (PhC) line-defect waveguides, topological edge states at an interface between, specifically, valley-Hall crystals of opposite K-valley pseudospin are usually *below* the light line of the slab, thereby decoupled from the radiation continuum.[33] As a result, in principle, these edge states do not intrinsically exhibit out-of plane losses, and only fabrication imperfections induce coupling to radiating modes. Under such conditions, in-plane backscattering becomes the dominant loss mechanism in the ultraslow-light regime (large values of n_g)—therefore, even *two-dimensional* PhC structures with the above topological implementation suffice to capture the physics of slow-light backscattering, which is not necessarily possible with other implementations of topological PhCs. Detailed calculations[33] have shown that topological valley-Hall photonic phases are, by approximately five times, more robust compared to standard PhC waveguides for small (technologically realistic) disorder levels—though this protection is, as expected, lost at higher levels of disorder. Large group refractive index values can, thus, be attained, of the order of $n_g \simeq 1000$, opening a promising route for strong and efficient light-matter interactions, where photon transmissions over hundreds of micrometers can be reasonably backscattering-free.

Concerning applications enabled by overcoming the limit, these would indeed be diverse, both, for classical and quantum devices, including but not limited to broadband low-loss nano-/micro-cavities,[1,14] enhancement of nonlinearities,[21] spontaneous emission rates and wave-matter interaction effects,[54,55] broadband room-temperature single-molecule strong coupling, broadband single-platform nanobiosensing[21,53] including chiral sensing,[18–20] all-optical buffers, broadband integrated spectrometers, single-photon sources with controllable spatiotemporal characteristics,[52] broadband invisibility cloaking of electrically-large objects (currently, a severely T-B limited scheme),[50,51] enhanced absorption for photovoltaics,[60–62] and improved nanoscale delivery of light for imaging and sensing,[17] to name only a few. All of these potential routes and applications are currently actively being researched, and it remains to be seen which one will eventually emerge and offer much required solutions to contemporary real-world applications and challenges.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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